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Social security and family support

Marie-Louise Leroux and Pierre Pestieau



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CORE DISCUSSION PAPER  
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**Social security and family support**

Marie-Louise LEROUX<sup>1</sup> and Pierre PESTIEAU<sup>2</sup>

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**Abstract**

This paper shows how the role of the market, the state and the family in providing financial support at old age has evolved over time with changes in factors such as the reliability and the effectiveness of family support, the rate of interest, the cost of public funds and earning inequality. We model a society in which agents with different productivity are asked to vote over the existence of a Beveridgian pension system. We show that when children assistance is certain and large, agents may rely exclusively on family to finance old-age consumption and prefer to vote for a zero tax rate. Only if income inequalities are very large, a majority will be in favor of a pension system. However, when the size and the likelihood of family generosity decreases, a pension system is more likely to emerge. In that case, agents supplement children assistance with pension benefits. A pension system is also more likely to emerge when the cost of public fund is small and the return from private savings is high.

**Keywords:** social security, hold-age security, family solidarity.

**JEL Classification:** D64, H55, J13

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# 1 Introduction

Social security represents one of the largest public expenditures of developed countries. For instance, it is estimated to account for 12.5% of the French GDP in 2007, while the average for OECD countries is around 7%.<sup>1</sup> Also, social security reform is one of the most fiercely debated topics in policy area. Thus, naturally this topic has attracted significant attention in public economics. However, a relatively under-researched question is: why does the degree of social security coverage differ so drastically across countries and societies? This paper attempts to tackle this question in a simple political economy framework that takes into account several ways of financing old-age consumption.

Most of the political economy literature which studies the size of social security highlights the conflicts of interest arising between different generations.<sup>2</sup> Some other papers also focus on the conflict of interest arising between agents belonging to a same cohort, but with different characteristics, such as longevity (see De Donder and Hindriks, 2002, Borck, 2007 and Leroux, 2010), productivity (Cremer et al. 2000a,b) or marital situations (see Leroux et al., 2011). All these papers model PAYGO pension systems and usually ignore the role of family ties. To the contrary, the objective of this paper is to consider explicitly intergenerational voluntary transfers within families along with the emergence of redistributive pension systems. In our model, there is only one generation of agents with different productivity. Parents invest resources in having children and in providing them with human capital.<sup>3</sup> In return, parents expect to receive transfers from children at old age, which may be determined either by children's altruism towards parents or by "implicit intergenerational contracts" or both. This point is illustrated in Table 1. Such intergenerational interactions provide individuals with a way of financing old-age consumption, which is indeed the prominent approach at least in "traditional" societies and almost everywhere in ancient times.

On the opposite, in "modern" societies, individuals can also maintain their living standard after retirement through two other main channels: saving made during working years, and social security that generally pursues two missions, that is forced saving and redistribution. In this paper, we develop a framework in which agents can rely on private saving, public pensions and

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<sup>1</sup>Source: OECD (2011).

<sup>2</sup>See for instance Browning (1975), Casamatta et al. (2000b).

<sup>3</sup>In this model, we do not distinguish between the number and the quality of children. This does not make any difference in the absence of direct utility obtained from having children. We come back to this point below.

children aid to finance old-age consumption.<sup>4</sup>

Country	%
Sundanese Indonesia	98
Javanese Indonesia	92
Philippines	89
Taiwan	79
Thailand	79
Turkey	77
Korea	54
Singapore	51
Germany	8
USA	8

Table 1: Old age security as a reason to have children (% women who answered "very important"), Source: Kagitcibashi, 1982.

As an illustration, Table 2 presents the share of these three sources of saving in old-age income.<sup>5</sup> It is clear that even within the European Community the respective role of the market, the state and the family in providing income for retirement varies quite a lot. Given that all these countries are well developed, the role of the family is reduced. Yet one sees some North-South gradient: as one goes from Nordic countries towards Mediterranean countries, the role of the family increases. As to the share between the market and the state, the Nordic and the Bismarckian countries tend to have relatively more generous public pensions than Beveridgean countries. As it will appear from our model, the existence of a pension system depends on a number of factors: the availability of financial market, the efficiency and reliability of family solidarity, the efficiency of the pension system and the distribution of earnings.

<sup>4</sup>We exclude here the possibility that retirees still have partial labour activity.

<sup>5</sup>We are grateful to J. Schoenmaeckers for this table.

Financial resources Countries	State	Market	Family	Family assistance	
				Time	Cash
Sweden	91	7.8	1.2	0.65	0.55
DanemarK	85.5	13.4	1	0.3	0.7
Netherlands	57	41.5	1.5	0.5	0.1
Belgium	94	2.5	3.5	0.6	2.9
Germany	88	7.5	4.5	0.5	4
Poland	94	0.5	5.5	0.5	5
Tchekia	95	2	3	1	2
Austria	92.5	3	4.5	0.9	3.6
Switzerland	68	30	1.5	0.5	1
France	95.5	2	2.5	0.5	2
Spain	92.5	3.5	4	0.3	3.7
Italy	92	4	4	1	3
Greece	84.5	6.5	9	2	7

Table 2: The role of the state, the market and the family in the old-age income. Source: own calculation from SHARE (2008).

In this paper, agents live for two periods and differ only with respect to productivity. We assume that they work in the first period and retire in the second one. They finance old-age consumption through private saving, public pensions and/or through investment in children. We use an old-age security model in which children are regarded as an asset, as opposed to dynastic households models in which parents care about their children (see Becker and Barro, 1988): agents derive utility only from consumption but not from having children and these are only seen as an old-age security device.<sup>6</sup> In the main part of the paper, we assume that family solidarity is certain but its generosity depends on some educational investment (and thus on income) and on a uniform family norm. The family norm accounts here for the intensity of family ties, which are likely to differ across societies and over time. Given investment possibilities (through the market, state and the family), agents vote for the existence of a redistributive pension system. We find that in societies where family solidarity is important (say traditional societies), a pension system is less likely to emerge. The median voter always prefers to rely exclusively on children, whose return is higher. It is only in the case where income inequalities are very large, that a majority of agents

<sup>6</sup>Adding a preference for both the quantity and the quality of children would not change our argument.

votes for a pension system and thus, prefer to supplement investment in children with public pensions. Quite the opposite, in modern societies, the magnitude of family help is likely to be lower and the return from children is likely to be dominated by other sources of investment, like public and private saving whose rate of return (i.e. the interest rate) is high. We also find that when a pension system is implemented, there is a substitution between family solidarity and the other sources of old-age income: both private saving and the number-quality of children are lower than when there is no pension system. Finally, in the last part of the paper, we allow for the possibility of uncertain family solidarity and show that in that case, agents never rely exclusively on children. Reliability is then a key factor: as soon as children can default, individuals seek income security in the financial market or in a social security scheme so as to ensure positive second-period consumption. We are back to a very standard political economy model. Since the median decisive voter obtains a higher return from public pensions than from private saving, a pension system is likely to emerge. The higher the probability of family default, the higher the equilibrium tax rate. All in all, we find that when family ties weaken (either through the size or the likelihood of family aid), a pension system is more likely to emerge.

Our paper is in line with the vast literature which studies investment in children as an old-age security device. Theoretical and empirical papers show that the observed decrease in fertility in modern societies may be partly due to the development of public pensions.<sup>7</sup> The key issue in that literature comes from the nature of the PAYGO mechanism. Indeed, if there is family solidarity, agents see the effect of their fertility decisions on the level of old-age benefits, while if solidarity is collective, agents have too low fertility rates as they do not correctly perceive the effect it has on pension returns. However, these papers do not explain why pension systems effectively emerge. In our paper, we assume that agents invest in children so as to obtain income in their old age, and the emergence of public pensions is the consequence of a decrease in family solidarity which we model through a decrease in the size of family aid and through the possibility of solidarity default. When a pension system emerges, fertility decreases. In a sense, compared to the existing literature, our paper goes a step behind: a decrease in family altruism (due to various reasons, such as geographic distance, change in social norms, etc.) should lead to the emergence of a pension system, which in turn leads to a decrease in fertility. Let us also mention the paper by Chakrabarti et al. (1993), which is one of the few papers that deal explicitly with the issue of family altruism

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<sup>7</sup>See for instance, Billari and Galasso (2009), Cigno and Rosati (1992), Galasso et al. (2009) and Zhang and Nishimura (1993).

default.<sup>8</sup> Contrary to our paper, theirs is normative and shows how uncertain altruism is likely to affect parents' investment in the human capital of their children. They also show that the intervention of a government through intergenerational transfers is likely to reduce the investment in human capital. Indeed, increasing the expected wealth of parents reduces the probability that they receive gifts from their children so that the expected return from children decreases and parents end up investing less in their children's human capital. This is in line with our results. Finally, also closely related to our paper is Belan and Wigniolle (2010) who study the political economy of pension systems when there is uncertain ascending altruism. Parents invest in the education of their children so as to instill them altruism. Altruism then accumulates over generations and after some threshold, a pension system becomes desirable as it is seen as a way to enforce old-age support from egoistic children. Thus, in their paper, a pension system emerges exactly for the opposite reason than in ours. One of the contributions of our paper is to put together those different strands of the literature.

The rest of the paper is organized as follows. In the next section, we present the model with no uncertainty as to children's assistance. In Sections 3 and 4, we derive individuals' preferred tax rates and the voting equilibrium. In Section 5, we show how our results are affected when children can default. The last section concludes.

## 2 The model

Let consider a population of agents, with mass 1, who have different productivity,  $w$ . We assume that productivity is a continuous variable with support  $[0, w_{\max}]$ , mean  $\bar{w}$  and median  $w_m < \bar{w}$ . For the following, we denote  $f(w)$  and  $F(w)$  as, respectively, the density and the cumulative functions of  $w$ .

The timing of the model is the following one. In the first period, individuals work and contribute to the pension system. Labour supply is exogenous and is normalised to 1.<sup>9</sup> They allocate their disposable income between first-period consumption  $c$  and private saving,  $s$ . They also have the possibility to invest in children for an amount  $e$  that represents the number as well as the quality of children an agent has.<sup>10</sup> Distinguishing between quality and

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<sup>8</sup>See also Sinn (2004) which considers PAYG pension systems as an enforcement device for ungrateful children.

<sup>9</sup>We could have as well assumed that labour supply depends on net income and thus on fiscal instruments. This would complicate the model without bringing more insights to the paper.

<sup>10</sup>In the paper, we usually refer to  $e$  as the "number of children" or as the "investment

quantity of children would certainly be more realistic but, in our case, this would not bring much but complications. For simplicity, we assume that the cost of a child, which includes both the cost of raising and educating him, is one. In the second period, individuals retire and their consumption, denoted by  $d$ , is equal to the pension benefit plus the gross return of their saving,  $(1 + r) s$ , where  $r$  is the interest rate. They also benefit from family solidarity and receive a compensation  $\phi(\alpha, e)$  from their children, such that

$$\begin{aligned}\phi_e(\alpha, e) &> 0, \phi_{ee}(\alpha, e) < 0 \\ \phi_\alpha(\alpha, e) &> 0, \phi_{\alpha\alpha}(\alpha, e) \leq 0 \\ \phi_{\alpha e}(\alpha, e) &> 0, \phi_e(\alpha, 0) = \infty.\end{aligned}\tag{1}$$

The parameter  $\alpha$  accounts here for an exogenous family norm and represents the intensity of family solidarity. For a given value of  $e$ , the level of compensation increases with  $\alpha$ . We also assume complementarity between  $\alpha$  and  $e$ :  $\phi_{\alpha e}(\alpha, e) > 0$ . In words the marginal return of an additional child,  $\phi_e(\alpha, e)$ , increases with the the social norm. In the main part of the paper, we assume that this compensation  $\phi(\alpha, e)$  is certain; only in a last section, we relax this assumption and show how it modifies our results. The last assumption,  $\phi_e(\alpha, 0) = \infty$ , makes sure that there will always be a strictly positive quantity-quality of children,  $e > 0$ .

The pension system is assumed to be Beveridgean and fully funded.<sup>11</sup> Note that considering a PAYGO system or a fully funded one does not really make a difference here since our model is static.<sup>12</sup> What really matters for individuals is the return obtained from the pension system, that is how much redistribution is operated. Workers contribute an amount  $\tau w$  in the first period where  $\tau$  is the payroll tax and in exchange, they receive a uniform pension benefit,  $P$ , at the time they retire, with  $P$  equal to

$$\begin{aligned}P &= (1 + \rho) \tau \int_0^{w_{\max}} w f(w) dw \\ &= (1 + \rho) \tau \bar{w}\end{aligned}\tag{2}$$

where  $\rho$  is the return before redistribution from the pension system and is in children" although, under our modeling, this also refers to quality.

<sup>11</sup>We could have as well taken a combination of Bismarck and Beveridge. This would not change our results.

<sup>12</sup>In a dynamic setting the effect of pensions on capital accumulation would have been relevant and then there is a big difference between a PAYGO system and a fully funded one. Another difference is that in a PAYGO system, the rate of return would be endogenous and related to  $e$ ; however in their choice the individuals do not take into account this endogeneity.

generally lower than the interest rate,  $\rho \leq r$ . This can be explained by some inefficiency of the public system resulting in a positive cost of public funds.

Agents derive utility only from consumption in the first and second periods of their life. Hence, under the assumption of no pure time preference, the utility of the agent can be represented by

$$U(c, d) = u(c) + u(d)$$

where the per period utility function  $u(\cdot)$  satisfies Inada conditions. We also assume that agents are liquidity constrained so that  $s \geq 0$ . In this very simple model, we assume that agents do not derive utility from having children or that they exhibit altruism toward them. Children are raised only for an old-age security motive.

### 3 Individuals preferred tax rates

#### 3.1 General analysis

In this section, we first study how agents with different productivity  $w$ , would like to finance their old-age consumption. We do so by finding individuals' preferences for the payroll tax rate, the level of saving and the number of children they choose to have. Replacing the first- and second-period budget constraints,

$$\begin{aligned} c + s + e &\leq w(1 - \tau) \\ d &\leq P + s(1 + r) + \phi(\alpha, e) \end{aligned}$$

into the utility function, the problem of an agent with productivity  $w$  consists in solving:

$$\begin{aligned} \max_{s, \tau, e} \quad & U(c, d) = u(w(1 - \tau) - c - s - e) + u(P + s(1 + r) + \phi(\alpha, e)) \\ \text{s.to } & s \geq 0, \tau \geq 0, e \geq 0 \end{aligned} \tag{A}$$

First-order conditions are such that:

$$\frac{\partial U}{\partial s} = -u'(c) + (1 + r)u'(d) \geq 0 \tag{3}$$

$$\frac{\partial U}{\partial \tau} = -wu'(c) + \bar{w}(1 + \rho)u'(d) \geq 0 \tag{4}$$

$$\frac{\partial U}{\partial e} = -u'(c) + \phi_e(\alpha, e)u'(d) \geq 0 \tag{5}$$

From the above first-order conditions, we study the preferences over  $(s, \tau, e)$  of an agent with productivity  $w$ . Note first that, under our assumptions on the function  $\phi(\alpha, e)$ ,

$$\partial U / \partial e|_{e=0} > 0$$

so that agents always have a strictly positive number-quality of children  $e(w)$ , which depends on their productivity and satisfies<sup>13</sup>

$$\phi_e(\alpha, e(w)) = \frac{u'(c)}{u'(d)}. \quad (6)$$

Second, whether the agent decides to supplement family solidarity with private saving or public pensions so as to finance his old-age consumption depends on the return he obtains from these two sources. Using (3) and (4), if the agent has a productivity such that  $w \leq \bar{w}(1 + \rho) / (1 + r)$ , he prefers public pensions to private saving so that  $\tau(w) \geq 0$  and  $s(w) = 0$ .

However, when agents have the possibility to finance their old-age consumption by investing in children, they also compare the return they obtain from a Beveridgean pension system with the return from family solidarity so that it might well be the case that some agents with productivity such that  $\phi_e(\alpha, e(w)) \geq \bar{w}(1 + \rho) / w$ , prefer to rely exclusively on children generosity. As public pensions redistribute resources from high productivity agents to low productivity ones, agents with higher productivity are less in favor of a pension system and prefer to invest more in children. Hence, for some intermediate range of productivity  $w \in [\hat{w}, \bar{w}(1 + \rho) / (1 + r)]$ , one may have that  $\tau(w) = s(w) = 0$  and  $e(w) > 0$ , where  $\hat{w}$  denotes the threshold after which agents prefer to rely exclusively on children. This threshold is implicitly defined by

$$\hat{w} = \frac{\bar{w}(1 + \rho)}{\phi_e(\alpha, e(\hat{w}))}$$

Finally, fully differentiating equation (6), it is clear that, over the interval  $[0, \bar{w}(1 + \rho) / (1 + r)]$ , the preferred number of children  $e(w)$  is an increasing function of  $w$ .

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<sup>13</sup>Consider instead an alternative modeling in which we distinguish between quality and quantity of children. Denote  $n$ , the number of children and  $h$ , their quality so that the return from children is now  $\phi(\alpha, n, h)$  and  $v(h, n)$  is the pure utility obtained from having  $n$  children with quality  $h$ . In the absence of pension system, the agent maximises the following utility function:

$$u(w - nh - s) + u(s(1 + r) + \phi(\alpha, n, h)) + v(h, n)$$

In our model, we make two simplifications:  $\phi(\alpha, n, h) = \phi(\alpha, e)$  with  $e = nh$  and  $\partial v / \partial h = \partial v / \partial n = 0$ . The second assumption is made so as to focus on the old-age security device, while the first one is made so as to simplify our notations.

The same kind of reasoning applies when the agent has a productivity  $w \geq \bar{w}(1 + \rho)/(1 + r)$ . In that case, he prefers private saving to public pensions,  $\tau(w) = 0$  and  $s(w) \geq 0$  but again, it might well be the case that he prefers to rely exclusively on children, if his productivity is such that the return obtained from children is greater than the one from private markets,  $\phi_e(\alpha, e(w)) \geq 1 + r$ . As  $\phi_e(\alpha, e(w))$  is decreasing in  $e(w)$  which is itself increasing in  $w$ , such an inequality is more likely to hold for lower productivity agents. Hence agents with productivity  $w \geq \bar{w}(1 + \rho)/(1 + r)$  but below some threshold, invest only in children and the higher their productivity, the higher is the number of children. At the threshold productivity  $\tilde{w}$ , the returns from saving and from children become equal,

$$\phi_e(\alpha, e(\tilde{w})) = 1 + r.$$

Agents with productivity  $w > \tilde{w}$ , invest a constant amount in children,  $\bar{e}$  such that  $\phi_e(\alpha, \bar{e}) = 1 + r$  and complete it with private saving. Over the interval  $[\tilde{w}, w_{\max}]$ ,  $s(w)$  is increasing with the agent's productivity.

Hence, depending on the productivity of the agent, the optimal division between public pensions, private saving and investment in children for financing second-period consumption is going to be different. However, it appears that this division crucially depends on the ranking between  $\bar{w}(1 + \rho)/(1 + r)$ ,  $\hat{w}$  and  $\tilde{w}$ , which at this stage can only be defined implicitly. In the next section, we specify functional forms so as to overcome this difficulty.

### 3.2 Closed form solution

In what follows we will assume the following functional forms:

$$\begin{aligned} u(x) &= \log(x) \\ \phi(\alpha, e) &= 2\alpha\sqrt{e} \end{aligned}$$

This is useful so as to obtain clearer results and go deeper in the political equilibrium analysis. Yet, our intuition is that our results would not be modified if we had specified another functional form for utility. Neither does it depend on the functional form of family solidarity, as soon as there is complementarity between  $\alpha$  and  $e$ , i.e.  $\phi_{\alpha e}(\alpha, e) > 0$ .

Combining the results of the previous section with our specification, the following proposition summarizes our findings:

**Proposition 1** *Preferences for the level of public pensions, private saving and children of an agent with productivity  $w$ , are such that, if  $u(x) = \log(x)$  and  $\phi(\alpha, e) = 2\alpha\sqrt{e}$ ,*

1. For an agent with  $w \leq \bar{w}(1 + \rho) / (1 + r)$ ,  $s(w) = 0$  and

$$e(w) = \begin{cases} \left(\frac{\alpha w}{\bar{w}(1+\rho)}\right)^2 & \text{if } w \leq \hat{w} \equiv \frac{1}{3} \left(\frac{\bar{w}(1+\rho)}{\alpha}\right)^2 \\ \frac{w}{3} & \text{if } w > \hat{w} \end{cases}$$

$$\tau(w) = \begin{cases} \frac{1}{2} - \frac{3}{2} \frac{\alpha^2 w}{[\bar{w}(1+\rho)]^2} & \text{if } w \leq \hat{w} \\ 0 & \text{if } w > \hat{w} \end{cases}$$

2. For an agent with  $w > \bar{w}(1 + \rho) / (1 + r)$ ,  $\tau(w) = 0$  and

$$e(w) = \begin{cases} \frac{w}{3} & \text{if } w \leq \tilde{w} \equiv 3 \left(\frac{\alpha}{1+r}\right)^2 \\ \left(\frac{\alpha}{1+r}\right)^2 & \text{if } w > \tilde{w} \end{cases}$$

$$s(w) = \begin{cases} 0 & \text{if } w \leq \tilde{w} \equiv 3 \left(\frac{\alpha}{1+r}\right)^2 \\ \frac{w}{2} - \frac{3}{2} \left(\frac{\alpha}{1+r}\right)^2 & \text{if } w > \tilde{w} \end{cases}$$

This proposition is formally proven in Appendix. When the agent has a low productivity, i.e.  $w \leq \bar{w}(1 + \rho) / (1 + r)$ , he chooses to finance old-age consumption through children and, possibly, through public pensions, but not through private saving. Under our functional forms, it is clear that within this interval of productivity, the higher the productivity, the smaller is the preferred tax rate and the higher is the number of children agents choose to invest in.<sup>14</sup>

Let us also mention that, in that case, the higher the return  $\rho$  from the pension system or equivalently its efficiency, the higher  $\tau(w)$  is and the higher is the level of  $\hat{w}$ , i.e. the threshold productivity after which agents prefer a zero tax rate. Hence, for a given productivity level  $0 < w \leq \hat{w}$ , the preference for the pension system increases when it becomes more interesting to invest in public pensions. On the other hand,  $d\tau(w)/d\alpha < 0$  and  $d\hat{w}/d\alpha < 0$  so that the intensity of the family norm goes exactly the other way round: if children become less generous, the return from family solidarity decreases and the preferred tax rate increases.

On the opposite, when the agent has a relatively high productivity, such that  $w > \bar{w}(1 + \rho) / (1 + r)$ , he will eventually supplement children investment with private saving but not with public pensions. The level of saving is clearly increasing in the agent's productivity.

As already mentioned, we need to compare  $\bar{w}(1 + \rho) / (1 + r)$ ,  $\hat{w}$  and  $\tilde{w}$ . This ranking is not obvious on prior grounds but, we prove in Appendix, that

<sup>14</sup>The tax rate is decreasing in  $w$  both because the utility from consumption exhibits a coefficient of relative risk aversion equal to 1 and because the number of children is increasing in  $w$ .

we have either  $\hat{w} < \bar{w}(1 + \rho) / (1 + r) < \tilde{w}$  or  $\tilde{w} < \bar{w}(1 + \rho) / (1 + r) < \hat{w}$  depending on whether

$$\bar{w} < 3 \frac{\alpha^2}{(1+r)(1+\rho)}$$

or  $\bar{w} > 3 \frac{\alpha^2}{(1+r)(1+\rho)}$

respectively. Whether one or the other case occurs depends on the average productivity level,  $\bar{w}$  but also, on the intensity of the family norm  $\alpha$ , on the returns from private saving  $r$  and from public pensions,  $\rho$ . Overall, the composition of second-period income will depend on which case is considered. Making use of the closed form solutions, we represent graphically these two cases and explain their implications below. To make it more intuitive, we will denote the first case T (for traditional societies) and the second case M (for modern societies).<sup>15</sup>

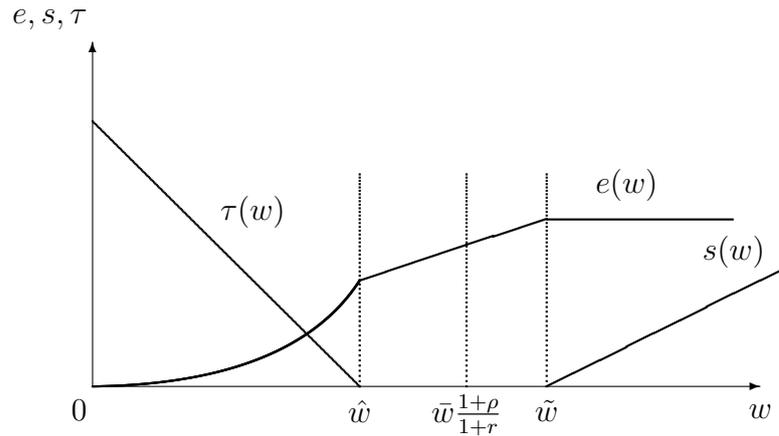


Figure 1: Case T,  $\bar{w} < 3 \frac{\alpha^2}{(1+r)(1+\rho)}$

<sup>15</sup>We realize that such a classification is rather restrictive. We simply want to emphasize that depending on the values of the parameters we would be more likely to be in one or the other type of society.

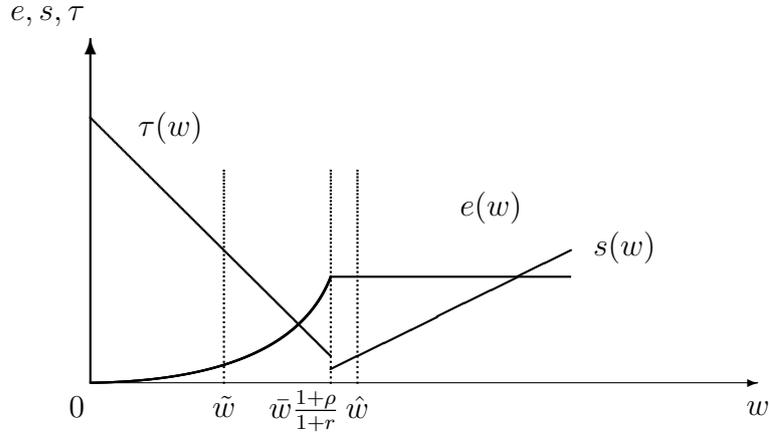


Figure 2: Case M,  $\bar{w} > 3 \frac{\alpha^2}{(1+r)(1+\rho)}$

To comment and interpret these figures, let us assume that average productivity,  $\bar{w}$  is fixed. Figure 1 then corresponds to the case of a traditional society, in which we would observe a high family norm, low return from saving and a high cost of public fund (i.e. low  $\rho$ ). Under our assumptions on  $\phi(\alpha, e)$ , agents always choose to invest in children so as to finance their old-age consumption. When they have low productivity,  $w < \hat{w}$ , they would also like to complement the investment in children with public pensions as they benefit from the redistribution operated through the Beveridgean pension system. On that interval, the tax rate is decreasing while the number of children is increasing with productivity, as both the return agents obtain from the pension system decreases and the total return from children,  $\phi(\alpha, e)$  increases. For some intermediate productivity level  $w \in [\hat{w}, \bar{w}(1+\rho)/(1+r)]$ , agents still get a positive return from the pension system, since  $w < \bar{w}(1+\rho)/(1+r)$ , but this is dominated by the return obtained from children so that they prefer to rely exclusively on family solidarity. On the contrary, when  $w > \bar{w}(1+\rho)/(1+r)$ , agents are now net contributors to the pension system so that they always prefer saving over public pensions. However, when they have a productivity level such that  $\bar{w}(1+\rho)/(1+r) < w < \tilde{w}$ , the marginal return from children  $\phi_e(\alpha, e(w))$  is higher than the one from private saving,  $(1+r)$  so that they prefer to rely exclusively on children's aid. As described previously,  $\phi_e(\alpha, e)$  is decreasing in  $e$  so that at the threshold productivity  $\tilde{w}$ , these returns become equal. Thus, agents with  $w > \tilde{w}$  invest a fixed amount in children,  $\bar{e} = (\alpha/(1+r))^2$  and decide to supplement it with saving. The higher their productivity, the larger their saving.

Figure 2 describes the modern society's situation in which the family norm is low, and / or the return from saving is high and the cost of public funds is low.<sup>16</sup> Again, agents always have a strictly positive number of children but, in this case, the return from these is low compared to the ones of public and private saving so that agents never rely exclusively on family solidarity. They always supplement it with either type of saving. This case fits our current societies. Whether they prefer one or the other source of old-age income depends on the relative return they obtain from it. If their productivity is low, then they obtain a higher return from public pensions than from private saving, i.e.  $\bar{w}(1 + \rho)/w > 1 + r$ . Otherwise they will prefer to invest in private saving.

Thus, whether intermediate productivity agents prefer to invest solely in children for financing their old-age consumption depends on the relative return of  $e$ . Indeed, in traditional societies, some middle class agents prefer to rely exclusively on children's aid, while this will never be the case in modern societies. One may then conjecture that the support for the pension system will be higher in modern than in traditional societies, as a consequence of a decrease in the intensity of the family norm and thus in the generosity of transfers from children.

Whether we are in one or the other situation will certainly influence the political equilibrium and the division of old-age income of agents between these three sources. We study these issues in the next section.

## 4 Majority voting equilibrium

In the first part of this section, we derive the equilibrium tax rate and show how it depends on the parameters  $(\alpha, \rho, r)$ . Depending on the majority-voting equilibrium, we show, in the second part of this section, what is the division between private, public saving and family solidarity in financing old-age consumption.

### 4.1 The political equilibrium tax rate

With single-peaked preferences, the median voter theorem applies and the political equilibrium tax rate  $\tau^*$  corresponds to the tax rate which is preferred by the median voter, so that  $\tau^* = \tau(w_m)$ .<sup>17</sup> As we mentioned in the previous

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<sup>16</sup>Note that in Figure 2, it is impossible to know exactly where  $\tau(w)$  and  $e(w)$  as well as  $s(w)$  and  $e(w)$  intersect. This depends on the values of the parameters  $(\alpha, \rho, r)$ .

<sup>17</sup>It is straightforward to show that the agent's utility is increasing and concave in the tax rate,  $\tau$ .

section, the political equilibrium and thus whether a majority of agents votes in favor of a pension system depends on the return they obtain from private saving, public pensions and family altruism, that is on the values of  $(\alpha, \rho, r)$ . Moreover, as we shall see, in some cases, the political equilibrium outcome also depends on wage inequality, that is on the gap between  $\bar{w}$  and  $w_m$ .

As before, we assume that  $\rho \leq r$ . Let us first consider the role of family norm on the equilibrium tax rate. When  $\alpha$  is high, it is more likely that  $\hat{w} < \bar{w} < \tilde{w}$  (case T) as  $d\hat{w}/d\alpha < 0$  and  $d\tilde{w}/d\alpha > 0$ . Since we also have that  $d\tau(w)/d\alpha < 0$ , it is straightforward to see that for high levels of  $\alpha$ , one can very well not have any pension or at best a very low one. This outcome can be mitigated if we have a wide gap between  $\bar{w}$  and  $w_m$ : indeed, only if the median productivity is much lower than the average one, we may observe the emergence of a pension system.

Let us then consider the role of private saving and of the interest rate. As  $r$  increases, it becomes more likely that  $\tilde{w} < \bar{w}(1 + \rho)/(1 + r) < \hat{w}$  and that we are in case M.<sup>18</sup> In that type of societies, and as long as  $\rho$  and  $r$  are not very different (we discuss that point below),  $w_m \leq \bar{w}(1 + \rho)/(1 + r)$  so that the median voter always votes for a strictly positive tax rate. The voting equilibrium tax rate is

$$\tau^* = \tau(w_m) = \frac{1}{2} - \frac{3}{2} \frac{\alpha^2 w_m}{[\bar{w}(1 + \rho)]^2} \quad (7)$$

From the above first-order conditions, we study the preferences over  $(s, \tau, e)$  of an agent with productivity  $w$ . Note first that, under our assumptions on the function  $\phi(\alpha, e)$ , The result that a pension system is more likely to emerge for higher  $r$  might seem surprising at first glance but this comes from the fact that both the returns from private saving and from public pensions are relatively high compared to  $\phi_e(\alpha, e)$ . Thus, whatever his productivity, it is always profitable for an agent to supplement the investment in children with either public pensions or private saving, the return from children being always dominated by the returns of these other two sources of old-age income. However as  $r$  increases and becomes very different from  $\rho$ , it may well be the case that  $w_m \geq \bar{w}(1 + \rho)/(1 + r)$  so that we may have  $\tau(w_m) = 0$ . In that case, the return from private savings is much higher than the one from the public pension so that the median voter would now prefer private saving over public pensions.<sup>19</sup> This case is quite extreme and rather unlikely in modern

<sup>18</sup>We have that  $d\tilde{w}/dr < 0$  and when  $r$  increases,  $\tilde{w}$  decreases more rapidly than  $\bar{w}(1 + \rho)/(1 + r)$ .

<sup>19</sup>The median voter invests  $e^*(w_m) = [\alpha/(1 + r)]^2$  and saves  $s^*(w_m) = \left[ w_m - 3 \left( \frac{\alpha}{1 + r} \right)^2 \right] / 2$ .

societies.

Finally, when the pension system becomes more efficient ( $\rho$  increases), it is more likely that case M arises.<sup>20</sup> Indeed, taxation is more efficient and the median voter will now be in favor of a positive tax rate (as it is clear from the above expression of  $\tau^*$ ).

These results remain true for more general utility and family solidarity functions, as long as the utility function is increasing and concave and  $\phi(\alpha, e)$  satisfies  $\phi_{\alpha, e} > 0$ . Our findings are summarized in the proposition below.

**Proposition 2** *Let assume that  $\rho \leq r$ . If  $u(x) = \log(x)$  and  $\phi(\alpha, e) = 2\alpha\sqrt{e}$ , the political equilibrium is such that:*

1. *A pension system is more likely to emerge*
  - *when the family norm parameter  $\alpha$  is low,*
  - *when income inequality ( $\bar{w} - w_m$ ) is high,*
  - *when the return from private saving  $r$  is high, but not too different from  $\rho$ ,*
  - *when the pension system is more efficient, i.e.  $\rho$  is high.*
2. *The generosity of the pension system is decreasing in  $\alpha$ , increasing in wage inequality, ( $\bar{w} - w_m$ ) and in  $\rho$  but invariant to the return from private saving,  $r$ .*

## 4.2 Equilibrium income division between private saving, public pensions and children

We are now interested in the relative role of the three sources of old-age income when the majority voting equilibrium tax rate is implemented. As described in Proposition 2 and in the previous subsection, two solutions are possible depending on the values of the parameters: either  $(\tau^*, P^*) = (\tau(w_m), \tau(w_m)\bar{w}(1+\rho))$  with  $\tau(w_m)$  defined by (7) or  $(\tau^*, P^*) = (0, 0)$ . Depending on whether a pension system emerges or not, agents choose the level of their private saving and the number of children they want to have, by solving problem (A) in which  $(\tau^*, P^*)$  are taken as given. Hence, relying on the first order conditions (3) and (5), as long as the marginal return from having children is greater than the interest rate,  $\phi_e(\alpha, e^*(w)) > (1+r)$ , that is when  $w$  is relatively low, the agent chooses to invest only in children and

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<sup>20</sup>We have that  $d\hat{w}/d\rho > 0$  so that when  $\rho$  increases,  $\hat{w}$  increases more rapidly than  $\bar{w}(1+\rho)/(1+r)$  and it becomes more likely that  $\tilde{w} < \bar{w}(1+\rho)/(1+r) < \hat{w}$ .

does not save. But beyond some productivity threshold defined by the equality of children and saving returns,  $\phi_e(\alpha, e^*(w)) = (1+r)$ , agents now choose to invest a constant amount in children and to supplement it with private saving to finance old-age consumption. These thresholds will be different depending on the value of  $(\tau^*, P^*)$  and are explicitly defined hereafter. Hence, using (3) and (5), we derive in the appendix, the division between  $e^*(w)$  and  $s^*(w)$  depending on the political outcome  $(\tau^*, P^*)$ :

**Proposition 3** *Assume that  $u(x) = \log(x)$  and  $\phi(\alpha, e) = 2\alpha\sqrt{e}$ . At majority voting equilibrium, private saving and investment in children are such that:*

1. *If the pension system is such that  $(\tau^*, P^*) = (\tau(w_m), \tau(w_m)\bar{w}(1+\rho))$ , an agent with productivity  $w$  chooses*

$$e^*(w) = \begin{cases} \frac{w(1-\tau^*)}{3} + \frac{P^*(P^* - \sqrt{P^{*2} + 12\alpha^2 w(1-\tau^*)})}{18\alpha^2} & \text{if } w \leq \check{w} \\ \left(\frac{\alpha}{1+r}\right)^2 & \text{if } w > \check{w} \end{cases}$$

$$s^*(w) = \begin{cases} 0 & \text{if } w \leq \check{w} \\ \frac{w}{2}(1-\tau^*) - \frac{1}{2(1+r)} \left[ \frac{3\alpha^2}{1+r} + P^* \right] & \text{if } w > \check{w} \end{cases}$$

where  $\check{w} \equiv \frac{3\alpha^2}{(1+r)^2(1-\tau^*)} + \frac{P^*}{(1+r)(1-\tau^*)}$

2. *If there is no pension system,  $(\tau^*, P^*) = (0, 0)$ , an agent with productivity  $w$  chooses*

$$e^*(w) = \begin{cases} \frac{w}{3} & \text{if } w \leq \tilde{w} \equiv 3\left(\frac{\alpha}{1+r}\right)^2 \\ \left(\frac{\alpha}{1+r}\right)^2 & \text{if } w > \tilde{w} \end{cases}$$

$$s^*(w) = \begin{cases} 0 & \text{if } w \leq \tilde{w} \equiv 3\left(\frac{\alpha}{1+r}\right)^2 \\ \frac{w}{2} - \frac{3}{2}\left(\frac{\alpha}{1+r}\right)^2 & \text{if } w > \tilde{w} \end{cases}$$

Note that  $\check{w}$  is the threshold productivity beyond which agents begin to save when a pension system is implemented and  $\tilde{w}$  is the same threshold when there is no pension system. Hence, when there is a pension system, agents with  $w \leq \check{w}$  only rely on children generosity and on public pensions, while those with a higher productivity level also privately save. The same comment applies when there is no pension system but in that case, the relevant threshold is  $\tilde{w}$  where it is possible to prove that  $\tilde{w} < \check{w}$ . In both cases,  $s(w)$  and  $e(w)$  are increasing in  $w$ .

Whether a pension system exists or not only influences the levels of  $s^*(w)$  and  $e^*(w)$ . To see this, let us use the above proposition and compare the relative role of private saving, public pensions and family solidarity, under the two regimes. Under our assumptions on the function  $\phi$ , agents always invest in children, whatever the political outcome. However, it is possible to show that the investment in children is always lower with a pension system than without.<sup>21</sup> In the same way, it is also straightforward to see from the above proposition that when a pension system is available, agents begin to save at higher productivity levels ( $\tilde{w} < \check{w}$ ) and for a given productivity, they save smaller amounts. Another way to say this is summarized in the proposition below.

**Proposition 4** *Under our specifications of  $u(\cdot)$  and  $\phi(\alpha, e)$ , the existence of a pension system crowds out investment in children and private saving.*

We finally briefly study second-period consumption:  $d^*(w) = s^*(w)(1+r) + P^* + \phi(\alpha, e^*(w))$  under the two political equilibrium outcomes. To do so, we resort to simulations as the outcome strongly depends on parameter values as well as on the distribution of productivity.<sup>22</sup> In either graph, we represent first a situation in which  $\rho = r$  and a pension system emerges and second, a situation in which  $\rho$  is smaller than  $r$  so that there is no pension system. We present our results for two levels of the family norm, which leads to either  $\tilde{w} < w_m$  or  $\tilde{w} > w_m$  (i.e. in the first case, the median agent also saves while in the second one, he relies only on family help, in the no pension case).<sup>23</sup>

<sup>21</sup>This can be shown by recognizing that  $de^*(w)/dw|_{P^*>0} < 1/3 = de^*(w)/dw|_{P^*=0}$ , that  $e^*(0)|_{P^*>0} = e^*(0)|_{P^*=0}$ , that  $\tilde{w} < \check{w}$  and that for any  $w \geq \tilde{w}$ ,  $e^*(w)|_{P^*>0} = (\alpha/(1+r))^2$  and for  $w \geq \check{w}$ ,  $e^*(w)|_{P^*=0} = (\alpha/(1+r))^2$ .

<sup>22</sup>Note that the two numerical illustrations below are robust to other parameter values.

<sup>23</sup>In graph (a), we assume that  $\alpha = 0.2$  while in graph (b),  $\alpha = 0.4$ . We also assume that  $w_m = 0.3 < \bar{w} = 0.5$  and  $r = 0.04$ . When  $\rho = r$ , the majority voting outcomes are  $(\tau^*, P^*) = (0.43, 0.225)$  for  $\alpha = 0.2$  and  $(\tau^*, P^*) = (0.23, 0.12)$  for  $\alpha = 0.4$ . When  $\rho = -0.7$  in both graphs, a majority of agents vote against the pension system.

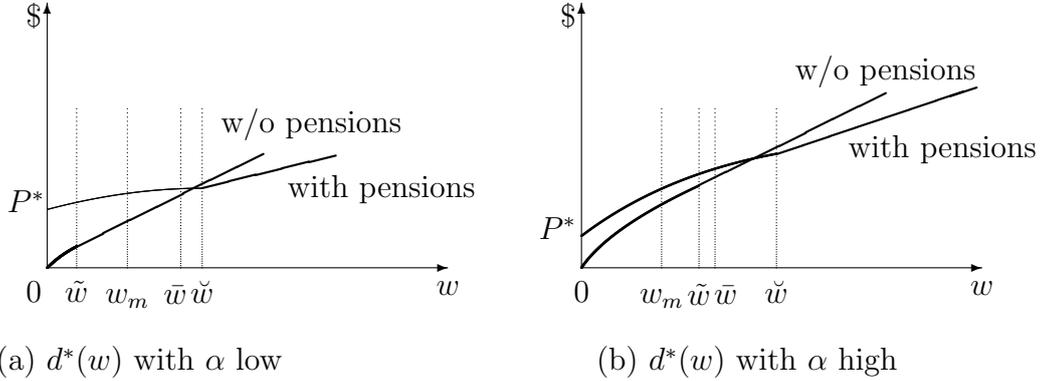


Figure 3: Second-period consumption

Figures 3 (a) and (b) show that, even if there is crowding out of private savings and investment in children, the presence of a pension system leads to higher old-age consumption for low-productivity agents than if there was no pension system. This result is independent of the level of the family norm,  $\alpha$ . This is also the case for agents with intermediate productivity, that is, those with productivity above  $\bar{w}$  but below the intersection between the curves. Only agents with high income (such that  $w$  is above the intersection) are better-off without pensions than with a pension system. Indeed, in a Beveridgean system, they do not benefit from redistribution, which decreases the amount of resources available for saving.

## 5 Uncertain family solidarity

In this section, we reconsider the specification of family solidarity and assume now it is uncertain. Indeed, in the previous sections, to model the importance of family solidarity and its variation over time, we only considered a variation in the intensity of the family norm, but not a change in its probability. We will now model the fact that investing in children may be risky and show how this affects our results. There may be many reasons why this probability is smaller than one, like migration, death of children...<sup>24</sup> We denote by  $p$ , the

<sup>24</sup>Here, uncertain family solidarity results from changes in socio-economic circumstances, but it is not linked for instance, to a change in fertility (and thus to individual decisions). It is also independent from the family norm.

probability of benefitting from family solidarity and we assume that it is the same for all agents. Second period consumptions are then equal to

$$\begin{aligned} d_1 &= P + s(1 + r) + \phi(\alpha, e) \\ d_2 &= P + s(1 + r) \end{aligned}$$

where  $d_1, d_2$  are second-period consumptions, respectively when there is family solidarity or not. Under this new specification, the problem of an agent with productivity  $w$  becomes

$$\begin{aligned} \max_{s, e, \tau} U(c, d) &= u(w(1 - \tau) - s - e) + pu(P + s(1 + r) + \phi(\alpha, e)) \\ &\quad + (1 - p)u(P + s(1 + r)) \\ \text{s. to } &s \geq 0, \tau \geq 0, e \geq 0 \end{aligned}$$

where  $P$  is defined by (2). First-order conditions for this problem are

$$\frac{\partial U}{\partial s} = -u'(c) + (1 + r)[pu'(d_1) + (1 - p)u'(d_2)] \geq 0 \quad (8)$$

$$\frac{\partial U}{\partial \tau} = -wu'(c) + \bar{w}(1 + \rho)[pu'(d_1) + (1 - p)u'(d_2)] \geq 0 \quad (9)$$

$$\frac{\partial U}{\partial e} = -u'(c) + p\phi_e(\alpha, e)u'(d_1) \geq 0 \quad (10)$$

As in the previous section, under our assumptions on  $\phi(\alpha, e)$ , agents always invest a positive amount in children  $e(w)$  defined by:

$$\phi_e(\alpha, e(w)) = \frac{u'(c)}{pu'(d_1)} \quad (11)$$

Let us now see how individuals decisions are modified under uncertain family solidarity. First, equation (11) is very similar to what we had in the certain probability case (see eq. 6), except for the probability  $p$ . As compared to the certain case, the number of children agents decide to invest in is still positive but reduced (if everything else is held constant). The lower is the probability of assistance from children, the lower the number of children agents decide to have. As before, the number of children increases with the productivity of the agent and with the intensity of the family norm.

Second, contrary to the certain probability case, we find that when  $p < 1$ , agents never invest solely in children but always supplement family aid with either private saving or public pensions, as a way to secure old-age income. Indeed, agents never rely exclusively on children transfers as it was the case previously, simply because in case of family default, they would be left with

no resource ( $d_2 = 0$ ). This decision is completely independent of the family norm and whether it is high or low. As it is clear from equations (8) and (9), their choice between private saving or public pensions depends on the relative return they obtain from each asset and thus on whether their productivity is greater or smaller than the threshold  $\bar{w}(1 + \rho) / (1 + r)$ . If the agent has a low productivity, that is if  $w \leq \bar{w}(1 + \rho) / (1 + r)$ , the return he obtains from public redistributive pensions is greater than the return he gets from private saving so that he is in favor of a pension system. On the contrary, for a productivity level  $w > \bar{w}(1 + \rho) / (1 + r)$ , he would be a net contributor to the pension system which redistributes from high-income toward low-income agents so that he prefers to invest in private saving and to vote for a zero tax rate. Our results are shown in Appendix and summarized in the proposition below.

**Proposition 5** *Under uncertain family solidarity, preferences for the level of public pensions, private saving and children of an agent with productivity  $w$ , are such that, if  $u(x) = \log(x)$  and  $\alpha(e) = 2\alpha\sqrt{e}$ ,*

1. *For an agent with  $w \leq \bar{w}(1 + \rho) / (1 + r)$ ,  $s(w) = 0$ . The tax rate and the number of children,  $\tau = \tau(w) > 0$  and  $e = e(w) > 0$ , satisfy the following system of equations*

$$\begin{cases} \left[ \frac{\bar{w}(1+\rho)}{\alpha} \sqrt{e} - w \right] \frac{1}{w(1-\tau)-e} + \frac{1-p}{\tau} = 0 \\ \frac{\alpha}{\sqrt{e}} - \frac{1}{p} \frac{\bar{w}(1+\rho)\tau + 2\alpha\sqrt{e}}{w(1-\tau)-e} = 0 \end{cases} \quad (12)$$

2. *For an agent with  $w > \bar{w}(1 + \rho) / (1 + r)$ ,  $\tau(w) = 0$ . The amount of private saving and number of children,  $s = s(w) > 0$  and  $e = e(w) > 0$ , satisfy the following system of equations*

$$\begin{cases} \left[ \frac{(1+r)}{\alpha} \sqrt{e} - 1 \right] \frac{1}{w-s-e} + \frac{1-p}{s} = 0 \\ \frac{\alpha}{\sqrt{e}} - \frac{1}{p} \frac{s(1+r) + 2\alpha\sqrt{e}}{w-s-e} = 0 \end{cases} \quad (13)$$

Note that, like in the certain altruism case, it is possible to show that whenever  $\tau(w) > 0$ , it always decreases with productivity.<sup>25</sup> This implies that the political equilibrium outcome exactly corresponds to the preferred tax rate of the median voter. Thus, when the probability of family solidarity is smaller than one, a pension system emerges if and only if  $w_m < \bar{w}(1 + \rho) / (1 + r)$ . In that case, the political outcome is  $(\tau^*, P^*) =$

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<sup>25</sup>To see that, replace (10) in (9) and differentiate it with respect to  $w$ .

$(\tau(w_m), \tau(w_m) \bar{w}(1 + \rho))$  where  $\tau(w_m)$  is the tax rate preferred by the median voter and solves the system of equations (12) evaluated at  $w_m$ . Such an outcome crucially depends on whether the pension system is efficient. If  $\rho \rightarrow r$ , it is very likely that  $w_m < \bar{w}(1 + \rho)/(1 + r)$  so that a majority of agents prefers to secure old-age income through public pensions. To the opposite, if the cost of public funds is relatively high, that is if  $\rho/r$  is low, it may be the case that  $\bar{w}(1 + \rho)/(1 + r) < w_m < \bar{w}$  so that a majority of agents prefers private saving to public pensions whose return is very small. In that case, no pension system emerges.

Comparing our results with what we had in Section 3 and 4 (and Figures 1 and 2), it is clear that whenever  $p < 1$ , a pension system is more likely to emerge as agents always need to secure old-age consumption, even if the family norm is very high. Here, when  $p < 1$ , the higher the risk of family default, the higher the equilibrium tax rate.<sup>26</sup> Hence, it appears that a decrease in the intensity of the family norm and / or in the probability of family solidarity makes more likely the emergence of a pension system.

Finally, let us mention that having different probabilities of family solidarity inside the society would not change this result. As long as some agents face uncertain family solidarity and that agents benefiting from certain family solidarity do not constitute a majority, a majority of agents need to secure old-age consumption. If  $w_m < \bar{w}(1 + \rho)/(1 + r)$ , most of them prefer to do it through public pensions. It will only affect the level of the equilibrium tax rate as it now depends on different  $ps$ .

## 6 Conclusion

In this paper, we explain the emergence of a Beveridgean pension system as the result of a switch from more traditional societies to modern societies. This switch is characterized by the development of financial markets, an increase in the efficiency of public institutions and by less constraining family norms. In the main part of our paper, family solidarity is certain and only the intensity of the norm varies, but we show in an extension of that model, that if family solidarity is uncertain, a pension system is more likely to emerge. We also show that when there is a pension system, agents have less children.

Our paper is in line with a number of studies on the old age security hypothesis. Accordingly, parents raise children because, when retired, they expect assistance from their children who are essentially seen as a capital good. Public pension programs and private saving by providing the same

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<sup>26</sup>Using the implicit function theorem on (9), it is possible to show that  $\tau$  is decreasing in  $p$ , for any  $w$ .

service as children can lead to an erosion of this motive for raising and educating children. This erosion of the old age security motive is often considered as a cause of the decline in fertility that triggered the demographic transition. Historically, we have indeed observed a decline in fertility but not in education. To account for the fact that we do not witness any decline in education, other factors would have to be introduced such as parental altruism or public provision of education: parents could prefer quality over quantity and the state could foster education as an engine of growth.

Finally, our model relies on a number of assumptions. Some are made for simplicity and do not influence our results, like for instance assuming a Beveridgean system and fixed labour supply. We also assume that the only reason for agents to have children and to educate them is old-age security, assuming away preferences for children and descending altruism. In fact, this latter assumption is in line with Cigno and Rosati (1992). Their paper compares theoretical predictions on the effects of financial markets and social security on fertility of two family choices models (one based on pure self-interest and the other on intergenerational altruism) with applications to the case of Italy. They found that an increase in social security coverage as well as wider capital market access and higher real interest rate have a significant negative impact on fertility decisions. These findings appear to favor the hypothesis that family decisions are more guided by pure self-interest than by family altruism.

Some other assumptions are more important, like for instance the assumption of a fully funded system. Assuming instead a PAYGO pension system would certainly change our results. In that case, the return from public pensions would be equal to the population growth rate which is likely to be high in traditional societies. Thus, agents would be in favor of a pension system. This support would even be reinforced in an overlapping generation model as the retirees would also be in favor of a free-lunch pension. In turn, this would make family solidarity less attractive, which in the end would lead to a decrease in fertility and to declining support for the pension system. These extensions are on our research agenda.

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## 7 Appendix

### 7.1 Proof of Proposition 1

Using the first-order conditions of problem (A), we obtain the following cases, which themselves can be divided into two subcases:

- If  $w \leq \bar{w}(1 + \rho) / (1 + r)$ ,  $s(w) = 0$  and  $\tau \geq 0$ . Using the above first-order conditions, we can also distinguish two subcases:

- if  $\phi_e(\alpha, e) \leq \bar{w}(1 + \rho) / w$ ,  $\tau > 0$  and satisfies

$$-wu'(c) + \bar{w}(1 + \rho)u'(d) = 0$$

which, jointly with the condition on  $e$ , yields that in equilibrium, one has

$$\frac{u'(c)}{u'(d)} = \phi_e(\alpha, e) = \frac{\bar{w}(1 + \rho)}{w}$$

Replacing with the functional forms of  $u(\cdot)$  and  $\phi(\alpha, e)$ , we get

$$\begin{aligned}\frac{d}{c} &= \frac{\alpha}{\sqrt{e}} = \frac{\bar{w}(1+\rho)}{w} \\ c &= w(1-\tau) - e \\ d &= \bar{w}\tau(1+\rho) + 2\alpha\sqrt{e}\end{aligned}$$

The solution to that system is

$$e(w) = \left(\frac{\alpha w}{\bar{w}(1+\rho)}\right)^2, \quad \tau(w) = \frac{1}{2} - \frac{3}{2} \frac{\alpha^2 w}{[\bar{w}(1+\rho)]^2}, \quad s(w) = 0$$

- On the contrary, if  $\phi_e(\alpha, e) > \bar{w}(1+\rho)/w$ , that is for  $w > \hat{w}$ ,  $\tau(w) = 0$  and  $e$  is such that

$$\begin{aligned}\phi_e(\alpha, e^*) &= \frac{u'(c)}{u'(d)} \\ &\Rightarrow \frac{\alpha}{\sqrt{e}} = \frac{d}{c}\end{aligned}$$

where  $d = \alpha 2\sqrt{e}$  and  $c = w - e$  so that  $e(w) = w/3$ . Replacing for the expression of  $e(w)$  in  $\phi_e(\alpha, e) \leq \bar{w}(1+\rho)/w$ , we have that the above solution is valid when  $w < \frac{1}{3} \left(\frac{\bar{w}(1+\rho)}{\alpha}\right)^2$ . For the following, we denote this threshold

$$\hat{w} = \frac{1}{3} \left(\frac{\bar{w}(1+\rho)}{\alpha}\right)^2$$

- If  $w > \bar{w}(1+\rho)/(1+r)$ ,  $\tau(w) = 0$  and  $s \geq 0$ .

- If  $\phi_e(\alpha, e^*) > (1+r)$ , the first-order condition for  $s$  is such that

$$\frac{\partial U}{\partial s} = -u'(c) + (1+r)u'(d) < 0$$

so that  $s(w) = 0$  and, as before,  $d = \alpha 2\sqrt{e}$  and  $c = w - e$ . Replacing in  $\phi_e(\alpha, e) = u'(c)/u'(d)$ , we obtain  $e(w) = w/3$ . Replacing for this expression into  $\phi_e(\alpha, e) > (1+r)$ , we obtain that these solutions are valid for any  $w \geq 3 \left(\frac{\alpha}{1+r}\right)^2$ . For the following, we denote this threshold,

$$\tilde{w} = 3 \left(\frac{\alpha}{1+r}\right)^2$$

– When  $\phi_e(\alpha, e^*) = (1+r)$ ,  $s(w) > 0$  and satisfies

$$-u'(c) + (1+r)u'(d) = 0$$

Jointly with the condition on  $e$ , we have

$$\frac{u'(c)}{u'(d)} = \phi_e(\alpha, e) = (1+r)$$

Again, replacing for the functional forms, we obtain

$$\begin{aligned} \frac{d}{c} &= \frac{\alpha}{\sqrt{e}} = (1+r) \\ c &= w - s - e \\ d &= s(1+r) + 2\alpha\sqrt{e} \end{aligned}$$

which yields the following solutions

$$e(w) = \left(\frac{\alpha}{1+r}\right)^2; s(w) = \frac{w}{2} - \frac{3}{2}\left(\frac{\alpha}{1+r}\right)^2$$

- As a third step, let us define the ranking between  $\bar{w}(1+\rho)/(1+r)$ ,  $\hat{w}$  and  $\tilde{w}$ . One has that  $\hat{w} < \bar{w}(1+\rho)/(1+r)$  if and only if

$$\begin{aligned} \frac{1}{3}\left(\frac{\bar{w}(1+r)}{\alpha}\right)^2 &< \bar{w}\frac{1+\rho}{1+r} \\ \Leftrightarrow \bar{w}\frac{1+\rho}{1+r} &< 3\left(\frac{\alpha}{1+r}\right)^2 \end{aligned}$$

where on the right-hand side, one recognizes  $\tilde{w}$ . Equivalently, when

$$\bar{w} < 3\frac{\alpha^2}{(1+r)(1+\rho)}$$

one has  $\hat{w} < \bar{w} < \tilde{w}$ . For the following, we will denote this case, case T. On the opposite, when

$$\bar{w} > 3\frac{\alpha^2}{(1+r)(1+\rho)}$$

one has  $\tilde{w} < \bar{w} < \hat{w}$ , which we will denote case M.

## 7.2 Proof of Proposition 3

We use (3) and (5) where

$$\begin{aligned} c &= w(1 - \tau^*) - s - e \\ d &= P^* + 2\alpha\sqrt{e} \end{aligned}$$

and  $(\tau^*, P^*)$  are either equal to  $(\tau(w_m), \tau(w_m)\bar{w}(1 + \rho))$  or to  $(0, 0)$ . If  $\phi_e(\alpha, e) > (1 + r)$ , the agent exclusively invests in children so that  $s^*(w) = 0$  and  $e^*(w)$  is defined implicitly by

$$\begin{aligned} \phi_e(\alpha, e^*(w)) &= \frac{u'(c^*)}{u'(d^*)} \\ \Rightarrow \frac{\alpha}{\sqrt{e^*(w)}} &= \frac{P^* + 2\alpha\sqrt{e^*(w)}}{w(1 - \tau^*) - e^*(w)} \end{aligned} \quad (14)$$

where we replaced for the functional forms on  $u(\cdot)$  and  $\alpha(\cdot)$ . When  $\phi_e(\alpha, e^*(w)) = (1 + r)$ , both  $s^*(w) > 0$  and  $e^*(w) > 0$  and satisfy the following system of equations:

$$\phi_e(\alpha, e^*(w)) = \frac{u'(c^*)}{u'(d^*)} = (1 + r)$$

which is equivalent to having,

$$\frac{P^* + 2\alpha\sqrt{e^*(w)} + s^*(w)(1 + r)}{w(1 - \tau^*) - e^*(w) - s^*(w)} = \frac{\alpha}{\sqrt{e^*(w)}} = 1 + r \quad (15)$$

We then solve equations (14) and (15) under the two possible majority voting equilibrium cases. When  $(\tau^*, P^*) = (\tau(w_m), \tau(w_m)\bar{w}(1 + \rho))$ ,

$$\begin{aligned} s^*(w) &= 0 \\ P^*\sqrt{e^*(w)} + 3\alpha e^*(w) - \alpha w(1 - \tau^*) &= 0 \end{aligned}$$

for any agent with  $w$  such that  $\phi_e(\alpha, e^*(w)) > (1 + r)$ . The above equation has two solutions:

$$e^*(w) = \frac{P^{*2} + 6\alpha^2 w(1 - \tau^*) \pm P^* \sqrt{P^{*2} + 12\alpha^2 w(1 - \tau^*)}}{18\alpha^2} \quad (16)$$

As the agent's utility increases in consumption which decreases with  $e$ , we retain the smallest one:

$$e^*(w) = \frac{P^{*2} + 6\alpha^2 w(1 - \tau^*) - P^* \sqrt{P^{*2} + 12\alpha^2 w(1 - \tau^*)}}{18\alpha^2} \quad (17)$$

We also have that  $de^*(w)/dw > 0$  with

$$\frac{de^*(w)}{dw} = \frac{1 - \tau^*}{3} \left[ 1 - \frac{P}{\sqrt{P^{*2} + 12\alpha^2 w (1 - \tau^*)}} \right] < \frac{1}{3}$$

and  $d^2e^*(w)/d^2w < 0$ . When  $\phi_e(\alpha, e^*(w)) = (1 + r)$ ,

$$\begin{aligned} e^*(w) &= \left( \frac{\alpha}{1 + r} \right)^2 \\ s^*(w) &= \frac{w}{2} (1 - \tau^*) - \frac{1}{2(1 + r)} \left[ \frac{3\alpha^2}{1 + r} + P^* \right] \end{aligned}$$

This is the case for

$$w > \frac{1}{(1 + r)(1 - \tau^*)} \left[ \frac{3\alpha^2}{1 + r} + P^* \right] \equiv \check{w}$$

Note that replacing for  $\check{w}$  in (17),  $e^*(\check{w}) = (\alpha/1 + r)^2$ . Replacing for  $(\tau^*, P^*) = (0, 0)$  in (14) and (15), it is straightforward to find back point 2 of Proposition 3.

### 7.3 Proof of Proposition 5

Using first-order conditions (8) and (9), two cases arise depending on whether  $w \leq \bar{w}(1 + \rho)/(1 + r)$ .

If  $w \leq \bar{w}(1 + \rho)/(1 + r)$ ,  $s(w) = 0$  and  $\tau(w) \geq 0$  satisfies

$$\frac{\partial U}{\partial \tau} = -wu'(c) + \bar{w}(1 + \rho)[pu'(d_1) + (1 - p)u'(d_2)]$$

Under the assumption that  $u'(0) \rightarrow \infty$ ,

$$\left. \frac{\partial U}{\partial \tau} \right|_{\tau=0} > 0$$

so that the solution for  $\tau(w)$  is always interior and satisfies

$$\left[ -w + \frac{\bar{w}(1 + \rho)}{\phi_e(\alpha, e(w))} \right] u'(c) + \bar{w}(1 + \rho)(1 - p)u'(d_2) = 0$$

where we substituted for (11). Replacing for functional forms in the above equation and in (11), we obtain point 1 of Proposition 5.

If  $w > \bar{w}(1 + \rho) / (1 + r)$ ,  $\tau(w) = 0$  and  $s(w) > 0$  satisfies

$$-u'(c) + (1 + r)[pu'(d_1) + (1 - p)u'(d_2)] = 0$$

Again under the assumption that  $u'(0) \rightarrow \infty$ ,  $s(w)$  cannot be null. Using (11), the above equation can be rewritten as

$$\left[ -1 + \frac{(1 + r)}{\phi_e(\alpha, e(w))} \right] u'(c) + (1 + r)(1 - p)u'(d_2) = 0$$

Replacing with the functional forms of  $u(\cdot)$  and  $\phi(\alpha, e)$  yields point 2 of Proposition 5.

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