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Credence goods and product differentiation

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Abstract

This paper analyses price competition between two firms producing horizontally and vertically differentiated goods. These are assumed to be credence goods, as consumers can hardly ascertain the quality of the commodities. We provide sufficient conditions for the existence of a unique price equilibrium and we characterize it. To illustrate the model, we adapt it to represent a newspapers' industry with two outlets, when the population of readers have preferences both on the political stance of the newspapers and on the accuracy of news they dispatch.

Keywords: credence goods, horizontal differentiation, vertical differentiation, newspaper industry.

JEL Classification: L1; L13, L15

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1 Introduction

A *credence good* is a commodity whose quality is difficult or impossible to ascertain by consumers, even after experimentation. For example, in the case of newspapers or magazines, readers may have difficulties in evaluating the accuracy with which the media outlets select and dispatch the news. Similarly, in the case of restaurants, consumers have difficulties in evaluating the quality and the freshness of the food, even after having experimented it.

Gabszewicz and Grilo (1992) and Bonroy and Constantatos (2008) have studied price competition between two firms selling vertically differentiated goods when consumers cannot ascertain which firm sells which quality. Their analysis is particularly suitable to understand price competition between credence goods. In the present paper, we expand upon Gabszewicz and Grilo (1992) by incorporating horizontal differentiation along with vertical one in a duopoly pricing model. We consider a model where goods are defined in a two characteristics space and consumers do not know which firm sells which quality. Introducing this specification leads to a model with two characteristics, the first one consisting of the horizontal characteristic and the second of the beliefs of consumers about firms´ product quality.

Several real life situations call for representing price competition within a model in which products are differentiated along several characteristics. Think of media outlets' competition. Daily newspapers or weekly magazines attract readers not only by the specificity of their content, would it be entertainment, culture, information or a mix of them, but also by the accurateness of the information content. Similarly, when consumers are planning to dine out and compare the merits of two restaurants, they consider the quality of the food, but also their location and their respective ambiance and surroundings. These examples illustrate the fact that consumers' preferences often reflect the interplay between horizontal and vertical aspects. Henceforth, preferences are generally not one-dimensional and utility comparisons must deal with more than one characteristic. While in the traditional literature, competition with differentiated products is basically analyzed using alternatively the two models of *horizontal* and *vertical* differentiation, some other contributions like Neven and Thisse (1990), Irmen and Thisse (1998), Levin, Peck and Ye (2009), Daughety and Reinganum, (2007) and (2008), or Gabszewicz and Wauthy (2011) have presented models that simultaneously embrace horizontal and vertical differentiation.

But often, even moving to models embedding multidimensional characteristics, as in Neven and Thisse (1990), is not sufficient enough to capture various significant real life ingredients of product differentiation. Among these, one of the most important is consumers' *uncertainty* about quality. Indeed, in several economic contexts, consumers are unable to assert unequivocally either the qualities of the variants offered in the market, or the firms who sell the fake or the high quality brand. For example, we often observe that when choosing between two media outlets, consumers highly value the rigor of the content provided by each media outlet, but they do not know with certainty which media outlet provides the most accurate content. Similarly, when consumers compare the merits of different restaurants, they ignore, or do not know with certainty, which restaurant uses the fresher ingredients. In fact, when analyzing market competition, imperfect information of consumers about product characteristics, and product quality in particular, is a recurrent problem.[1]

To the best of our knowledge, only a few papers have presented models simultaneously embracing horizontal and vertical differentiation with private information about firms' quality. Levin, Peck and Ye (2009) investigate how information disclosure can be used by firms to manipulate consumers' beliefs within a model in which consumers observe firms' location (which is the source of horizontal differentiation) while being uninformed about product's quality. Daughety and Reinganum (2007) and (2008) also present models in which goods are both horizontally and vertically differentiated and firms have private information about the quality of their products. These models investigate price competition when prices can signal the quality of the goods.

The present paper contributes to the recent literature involving simultaneously horizontal and vertical differentiation with private information about firms' quality by providing a different approach to uncertainty about product quality. While there are indeed many cases in which the observable choices made by firms are able to provide consumers with a signal about products' quality (in line with the papers mentioned before), there are other situations in which it is hard to use signalling devices to alleviate informational problems.

A particularly relevant type of goods corresponding to the last case are the credence goods, in which consumers cannot ascertain the quality of the good through experimentation. As pointed out by Bonroy and Constantatos (2008), with credence goods, the production of a bad quality cannot be detected and punished and therefore signalling becomes very hard since the delayed detection of the low quality good allows its producer to mimic the strategy of the good quality firm.[2] Accordingly, in the case of credence goods, consumers can only choose between the available goods on the basis of subjective beliefs about the quality of each product. Going back to the examples above, we confirm that both media outlets and restaurants may be viewed as credence goods. In the case of media outlets, we observe that consumers may be willing to pay a premium for more accurate news but they may have difficulties in distinguishing which journal has more accurate news. Similarly, in the case of restaurants, consumers may be willing to pay a premium for the use of healthier ingredients (e.g. biological products *versus* genetically modified products). However, even after repeated experimentation, it remains very difficult to ascertain the quality of the ingredients used in the food served in each restaurant.

[1]There exists a huge literature devoted to market behavior under uncertainty (see, e.g., Shapiro (1983), Wolinsky (1986), Ungern-Sternberg and Weizacker (1985), Salop and Stiglitz (1977), Stahl (1982)).

[2]The existence of credence goods leads their sellers to give advice to consumers about the relative quality of the goods, which gives rise to a situation of asymmetric information. This information asymmetry creates obvious incentives for an opportunistic behavior by the sellers. Wolinsky (1995) studies how these phenomena affect the functioning of credence goods markets.

In light of these examples, the present paper proposes a model combining horizontal and vertical differentiation when consumers do not know which firm sells which quality, and firms are not able to rely on signalling devices to disclose the true quality of their goods. Our model is based on a synthesis of two papers. The first is by Neven and Thisse (1990) and the second by Gabszewicz and Grilo (1992). From the first, we borrow the idea of constructing a model which accounts for *two-dimensional* competition, one related to a horizontally differentiated characteristic of the good, and the other to a vertically differentiated component. From the second one, we borrow the idea of relating vertical product differentiation to uncertainty about the quality of the goods when firms cannot manipulate consumers' beliefs through signalling devices (e.g. credence goods). Following their approach, we assume that consumers do not know which firm sells which quality, holding heterogeneous beliefs about this event. Consumers' beliefs tend to be based on all the information made available to them, like, for example, press reports, word-of-mouth, and so on. In light of this fact, consumers' beliefs are often heterogeneous since consumers' ability to search, absorb or process the available information may obviously differ among consumers[3].

In the context of our game of static price competition between firms selling horizontally and vertically differentiated products (whose quality is uncertain), we characterize the interior equilibria in prices, providing sufficient conditions for its existence and uniqueness. We show that the characteristics of the equilibrium are different depending on whether the dispersion of beliefs is more significant than the dispersion of tastes, or *vice-versa.*

In addition, we provide an application of the model to the newspaper industry, in which readers' preferences over media outlets depend *(i)* on the political viewpoints of the newspaper, that represent a horizontal differentiation dimension, as well as on *(ii)* the accuracy with which news are selected and dispatched by the newspapers, which introduces a vertical differentiation element in readers' preferences. As the newspapers' degree of accuracy is hard to ascertain even after reading the news, in the same spirit of the abstract model, we consider that readers are uncertain about which newspaper is the most credible one. In the context of this application, we show that equilibrium prices depend on whether the dispersion in political opinions is more significant (or not) than the dispersion of beliefs regarding the newspapers' credibility. In equilibrium, the prices of the newspapers balance the popularity of the newspapers' political ideology and the credibility of their information. The weight assigned to each of these elements depends on the relative dispersion of political opinions *vis-à-vis* the dispersion of beliefs regarding the accuracy of the news in each newspaper. Relying on a numerical example, we illustrate how an exogenous shock in the newspapers' credibility may affect in different ways the nature of price competition between the newspapers.

[3]Population's beliefs about which firm sells which quality can make some consumers to view one of the firms, say firm 1, as being more than likely the seller of the high quality variant, while other consumers may well believe the reverse, inverting thereby their preferences for firm 1 at the advantage of firm 2.

The paper is organized as follows. Section 2 presents the model. Section 3 provides the corresponding price equilibrium analysis. Section 4 develops an application of the model to investigate price competition in the newspaper industry. Finally, section 5 concludes.

2 The model

We consider two profit maximizing firms (firm 1 and firm 2) that produce two goods (good 1 and good 2), differentiated along two characteristics. Goods are produced at a constant marginal cost assumed to be equal to zero, without loss of generality. With respect to the first characteristic, products 1 and 2 differ by their *quality*. However, as in Gabszewicz and Grilo (1992), consumers are uncertain about which firm sells which quality and, furthermore, consumers differ in their beliefs about this uncertain event. The goods are supposed to be differentiated horizontally with respect to the second characteristic which is perfectly observable. Thereby, our model accounts for *two* distinct sources of heterogeneity among consumers: *(i)* consumers formulate different subjective probabilities (beliefs) concerning which good has a higher-quality in terms of the unobservable characteristic; and *(ii)* they differ on their evaluation of goods' merit in relation to the observable characteristic.

Each consumer is identified by a vector (m, θ). The first component (m) represents the subjective probability that consumer of type m assigns to the event: $\{$*product* 1 *corresponds to the high-quality product*$\}$. We assume that m is uniformly distributed in the interval $[\underline{m}, \overline{m}]$, with

$$0 \leq \underline{m} < \frac{1}{2} < \overline{m} \leq 1.$$

We call the interval $[\underline{m}, \overline{m}]$ the *domain of beliefs*. The second component (θ) deals with the heterogeneity of consumers with respect to the horizontal differentiation characteristic, measuring the differential in utilities consumer θ gets when he/she consumes good 1 *versus* good 2. We assume that θ is uniformly distributed in $[\theta_{\min}, \theta_{\max}]$, with $\theta_{\min} < 0$, representing the consumer who, concerning this characteristic, prefers the most good 2 to good 1; and $\theta_{\max} > 0$, representing the consumer who prefers the least good 2 to good 1 regarding this same characteristic.

The set of consumers (m, θ) is represented in the rectangle $[\underline{m}, \overline{m}] \times [\theta_{\min}, \theta_{\max}]$. When consumer (m, θ) buys good 1, at price p_1, she/he expects to obtain a (expected) utility $E_{U_1}(m, \theta, p_1)$ defined by

$$E_{U_1}(m, \theta, p_1) = V + m(u_h) + (1 - m)(u_l) + \frac{\theta}{2} - p_1, \tag{1}$$

where V denotes a positive constant[4], and u_h (resp. u_l) is the utility provided by the variant which corresponds to the high-quality (resp. low-quality) product

[4]The constant V is considered to be large enough for all consumers to find a product for which their utilities are positive at equilibrium (covered market).

with respect to the unobservable characteristic,with $u_h > u_l \geq 0$. When consumer (θ, m) buys good 2 (instead of good 1), she/he expects to get a (expected) utility $E_{U_2}(m, \theta, p_2)$ defined by

$$E_{U_2}(m, \theta, p_2) = V + m(u_l) + (1-m)(u_h) - \frac{\theta}{2} - p_2. \tag{2}$$

Comparing (1) and (2), it becomes clear that the parameter θ indeed represents the differential in utilities (with respect to the horizontal differentiation component) consumer θ gets when she/he consumes good 1 *versus* good 2. For example, everything else the same, consumer of type $\theta_{\max}$ obtains an increase in utility of $\frac{\theta_{\max}}{2}$ when consuming good 1, and a decrease in utility of $\frac{\theta_{\max}}{2}$ when consuming good 2. On the contrary, consumer of type $\theta_{\min}$ gets a utility reduction of $\frac{\theta_{\min}}{2}$ when consuming good 1 and a utility increase of $\frac{\theta_{\min}}{2}$ when consuming good 2.

Conditional on prices (p_1, p_2) and for a given belief $m \in [\underline{m}, \overline{m}]$,the consumer $\left(\tilde{\theta}(m), m\right)$ satisfying the equality

$$E_{U_1}\left(m, \tilde{\theta}, p_1\right) = E_{U_2}\left(m, \tilde{\theta}, p_2\right) \Leftrightarrow \tag{3}$$

$$\Leftrightarrow \tilde{\theta}(m) = p_1 - p_2 + (1-2m)(u_h - u_l) \tag{4}$$

is indifferent between buying good 1 or good 2.

From expression (4) it follows that, for a given vector of prices $(p_{1,}p_2)$, the marginal consumer $\tilde{\theta}(m)$ evolves linearly and negatively with m :

$$\frac{\partial \tilde{\theta}}{\partial m} = -2(\Delta u) < 0,$$

with $\Delta u = u_h - u_l$. Not surprisingly, for those types of consumers that assign a greater probability m to the event {*product* 1 *corresponds to the high-quality product*}, the critical value of $\tilde{\theta}(m)$ is lower. Thus, the higher the $m-$value, the greater the mass of consumers θ who are willing to buy good 1 instead of good 2.

The following figures represent the set of consumers (m, θ) in the rectangle $[\underline{m}, \overline{m}] \times [\theta_{\min}, \theta_{\max}]$ and the corresponding linear function $\tilde{\theta}(m)$ in two different cases, according to the values of the parameters.[5]

In both cases, the consumers lying on the line $\tilde{\theta}(m)$ are indifferent between buying the two goods, when m varies over $[\underline{m}, \overline{m}]$. Those above the line $\tilde{\theta}(m)$

[5] Notice that Figure 1 and Figure 2 do not cover all the possible demand configurations corresponding to our problem. The full analysis of demand is quite tedious and we send it to Appendix A.

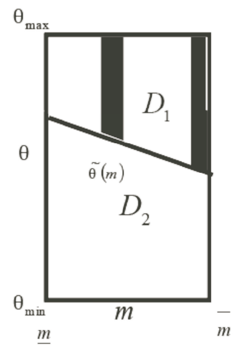

Fig.1 - Tastes' dispersion dominates the dispersion of beliefs

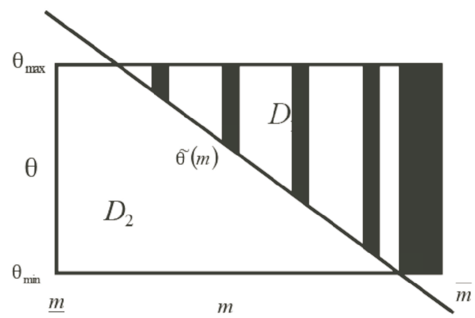

Fig. 2 - Beliefs' dispersion dominates dispersion of tastes

buy good 1, while those below buy good 2. Notice that, in figure 1, the dispersion of tastes over the horizontal characteristic is more significant than the dispersion of beliefs regarding which of the two goods is of higher quality. On the contrary, on figure 2, the reverse holds and the dispersion of beliefs is more significant than the dispersion of consumers' tastes over the horizontal characteristic of the good. Accordingly, the demand functions for the two goods depend on whether tastes dispersion dominates the dispersion of beliefs, or *vice-versa*[6]. It is easy to show that the first situation holds if and only if, the inequality

$$2\Delta u < \frac{\theta_{\max} - \theta_{\min}}{\overline{m} - \underline{m}} \tag{5}$$

is satisfied. The proof behind condition (5) is left to Appendix B.

It is easily seen that, in both cases, demands are continuous and decreasing in firms' own prices. However, there are domains of parameters in which demands

[6]A similar situation arises in the paper by Neven and Thisse (1992), in which the configurations of demand depend on the relative weight consumers put on the horizontal *versus* the vertical dimension of their preferences. They introduce the terminology of *horizontal* and *vertical dominance* to identify the two resulting configurations of demand.

are not everywhere concave functions[7]. Nevertheless, sufficient conditions can be defined to guarantee that, in these domains, individual profits are decreasing in firms' own prices, for a given rival's price. These conditions are thus sufficient to prevent the occurrence of unilateral advantageous deviations which could lead a pair of prices to fall in the above mentioned domains. Lemma 1 and Lemma 2 in Appendix respectively present these sufficient conditions when $2\Delta u < \frac{\theta_{\max}-\theta_{\min}}{\overline{m}-\underline{m}}$, or vice-versa.

In the following section we characterize equilibrium prices. Firstly, in subsection 3.1 we analyze equilibrium prices when the dispersion of tastes is more significant than the dispersion of beliefs. Then, in subsection 3.2, we investigate the opposite case.

3 Equilibrium analysis

In this section, we characterize the Nash equilibrium arising in a static game, in which *(i)* firms set their prices simultaneously and non-cooperatively; *(ii)* firms have complete information, whereas consumers are uncertain about product quality; and *(iii)* consumers' preferences correspond to the ones described in the previous section. As the demand functions for the two goods depend on whether tastes dispersion dominates the dispersion of beliefs, or *vice-versa,* the equilibrium analysis has to be performed separately for the two cases. When $2\Delta u < \frac{\theta_{\max}-\theta_{\min}}{\overline{m}-\underline{m}}$ and the conditions in lemma 1 hold[8], firms' profits are quasi-concave functions of prices and they reach only one maximum value in the region[9] where the profit functions write as

$$\pi_1^H\left(p_1,p_2\right)=\left(\frac{\theta_{\max}-p_1+p_2+\left(\underline{m}+\overline{m}-1\right)\Delta u}{\theta_{\max}-\theta_{\min}}\right)p_1,$$

for firm 1, and

$$\pi_2^H\left(p_1,p_2\right)=\left(1-\frac{\theta_{\max}-p_1+p_2+\left(\underline{m}+\overline{m}-1\right)\Delta u}{\theta_{\max}-\theta_{\min}}\right)p_2,$$

for firm 2.

Proposition 1 *When $2\Delta u < \frac{\theta_{\max}-\theta_{\min}}{\overline{m}-\underline{m}}$ and the conditions in lemma 1 hold, there exists a unique price equilibrium, given by:*

$$\begin{aligned}p_1^{H*} &= \tfrac{1}{3}\left(\left(\theta_{\max}-\theta_{\min}\right)+\theta_{\max}+2\left(\tfrac{\underline{m}+\overline{m}}{2}-\tfrac{1}{2}\right)\Delta u\right),\\ p_2^{H*} &= \tfrac{1}{3}\left(\left(\theta_{\max}-\theta_{\min}\right)-\theta_{\min}-2\left(\tfrac{\underline{m}+\overline{m}}{2}-\tfrac{1}{2}\right)\Delta u\right),\end{aligned} \tag{6}$$

with $p_1^{H*}>0$ and $p_2^{H*}>0$.

[7] It can be seen that the demand of firm 1 is convex in $(p_1,p_2)\in R_2^j, j=H,V$ where the expression of $R_2^j, j=H,V$ is defined as in Appendix A. Analogously, demand of firm 2 is convex in $(p_1,p_2)\in R_4^j, j=H,V$ where again $R_4^j, j=H,V$ is defined as in Appendix A. In the remaing regions, the demand functions can be easily shown to be concave.

[8] See the Proof of Lemma 1 in Appendix B.

[9] For more details see appendix A, where this region is referred as R_3^H.

Proof. See Appendix B. ■

Now, when $2\Delta u > \frac{\theta_{\max}-\theta_{\min}}{\overline{m}-\underline{m}}$ and the conditions in lemma 2 hold[10], firms' profits are quasi-concave functions of prices and they reach only one maximum value in the region[11] where the profit functions write as

$$\pi_1^V(p_1, p_2) = \left(\frac{1}{4}\frac{\theta_{\max}+\theta_{\min}-2p_1+2p_2-2(1-2\overline{m})\Delta u}{\Delta u(\overline{m}-\underline{m})}\right)p_1,$$

for firm 1, and

$$\pi_2^V(p_1, p_2) = \left(1-\frac{1}{4}\frac{\theta_{\max}+\theta_{\min}-2p_1+2p_2-2(1-2\overline{m})\Delta u}{\Delta u(\overline{m}-\underline{m})}\right)p_2,$$

for firm 2.

Proposition 2 *When $2\Delta u > \frac{\theta_{\max}-\theta_{\min}}{\overline{m}-\underline{m}}$ and the conditions in lemma 2 hold, there exists a unique price equilibrium, given by:*

$$\begin{array}{l} p_1^{V*} = \frac{1}{3}\left(\left(2\left(\overline{m}+\left(\overline{m}-\underline{m}\right)\right)-1\right)\Delta u + \frac{\theta_{\max}+\theta_{\min}}{2}\right), \\ p_2^{V*} = \frac{1}{3}\left(\left(2\left(\left(\overline{m}-\underline{m}\right)-\underline{m}\right)+1\right)\Delta u - \frac{\theta_{\max}+\theta_{\min}}{2}\right), \end{array} \tag{7}$$

with $p_1^{V*} > 0$ and $p_2^{V*} > 0$.

Proof. See the Appendix B. ■

From Proposition 1 and 2, it follows that under perfect symmetry of tastes (i.e. $\theta_{\max} = -\theta_{\min}$) and beliefs (i.e. $\frac{\underline{m}+\overline{m}}{2} = \frac{1}{2}$), the two equilibrium prices are equal. By contrast, it is the asymmetry in tastes and/or in beliefs which generates a price differential between the two goods. For instance, if $\theta_{\max} = -\theta_{\min}$ and good 1 is more trustworthy than good 2, we get $2\Delta u > \frac{\theta_{\max}-\theta_{\min}}{\overline{m}-\underline{m}}$ and $p_1^{V*} > p_2^{V*}$, reflecting this advantage of good 1 over good 2.

Comparing our results with Gabszewicz and Grilo (1992), we observe that the equilibrium prices in the latter coincide with ours when we "neutralize" the heterogeneity of consumers' tastes with respect to the horizontal characteristic, i.e. $\theta_{\max} = -\theta_{\min} = 0$. In that case, the inequality $2\Delta u > \frac{\theta_{\max}-\theta_{\min}}{\overline{m}-\underline{m}}$ necessarily holds. When $\theta_{\max} > -\theta_{\min}$ (resp. $\theta_{\max} < -\theta_{\min}$), the equilibrium price of firm 1 (resp. firm 2) exceeds the corresponding equilibrium price of this firm in Gabszewicz and Grilo (1992), while the other firm charges a lower price in our setting. In the case $2\Delta u < \frac{\theta_{\max}-\theta_{\min}}{\overline{m}-\underline{m}}$, "neutralizing" the heterogeneity of consumers' preferences with respect to the horizontal characteristic ($\theta_{\max} = -\theta_{\min} = 0$) necessarily requires Δu to be equal to zero[12]. In that case, both firms quote prices equal to zero at equilibrium: since there is no more any source of differentiation between goods, we end up with pure competition "à la Bertrand".

[10] See the Proof of Lemma 2 in Appendix B.

[11] For more details see appendix A, where this region is referred as R_3^V.

[12] This corresponds to a degenerate case of lemma 1 in the Appendix. When $\theta_{\max} = -\theta_{\min} = 0$, given that $\underline{m} < \frac{1}{2} < \overline{m}$, the conditions in lemma 1 are only valid for $\Delta u = 0$.

Now assume that quality is perfectly observable and, without loss of generality, good 1 is the high-quality good. Then $\overline{m} = \underline{m} = 1$ and the only source of heterogeneity among consumers comes from the horizontal differentiation component (θ).

Then, the demand for good 1 obtains as:

$$D_1(p_1, p_2) = \begin{cases} 0 & if \quad p_1 - p_2 > \theta_{\max} + \Delta u \\ \frac{\theta_{\max} - (p_1 - p_2 - \Delta u)}{\theta_{\max} - \theta_{\min}} & if \quad \theta_{\min} + \Delta u < p_1 - p_2 < \theta_{\max} + \Delta u \\ 1 & if \quad p_1 - p_2 < \theta_{\min} + \Delta u \end{cases} \tag{8}$$

and

$$D_2(p_1, p_2) = 1 - D_1(p_1, p_2). \tag{9}$$

It is easily seen that firms' demands are continuous, decreasing and concave functions of firms' own prices. Firms' best-reply functions are then given by[13]:

$$p_1(p_2) = \begin{cases} \frac{1}{2}(\theta_{\max} + p_2 + \Delta u) & if \quad 0 < p_2 < (\theta_{\max} - 2\theta_{\min} - \Delta u) \\ p_2 + \theta_{\min} + \Delta u & if \quad p_2 > (\theta_{\max} - 2\theta_{\min} - \Delta u) \end{cases},$$

in the case of firm 1, and

$$p_2(p_1) = \begin{cases} p_1 - \theta_{\max} - \Delta u & if \quad p_1 > 2\theta_{\max} - \theta_{\min} + \Delta u \\ -\frac{1}{2}(\theta_{\min} - p_1 + \Delta u) & if \quad \theta_{\min} + \Delta u < p_1 < 2\theta_{\max} - \theta_{\min} + \Delta u \\ p_1 - \theta_{\min} - \Delta u & if \quad p_1 < \theta_{\min} + \Delta u \end{cases},$$

in the case of firm 2.

Thus, in a scenario of perfect information, equilibrium prices with both firms active in the market obtain as:

$$p_1^{PI*} = \tfrac{1}{3}(2\theta_{\max} - \theta_{\min} + \Delta u),$$
$$p_2^{PI*} = \tfrac{1}{3}(\theta_{\max} - 2\theta_{\min} - \Delta u),$$

which occurs as long as:

$$\theta_{\max} - 2\theta_{\min} > \Delta u.$$

Proposition 3 *Under perfect information, when the conditions of lemma 1 hold, both firms are active at equilibrium. Furthermore, equilibrium prices coincide with those obtained for imperfect information when the domain of beliefs is degenerate and reduces to the singleton* $m = 1$.

Accordingly, the introduction of a *"perfect label"*, that would perfectly disclose which firm sells which quality, leads to an increase in the price of the high quality good and a concomitant decrease in the price of the low-quality one.[14]

[13] We do not consider price policies leading to the eviction of firm 2, since these would require $p_2 < -\theta_1 - \Delta u$, which is inconsistent with the non-negativity constraint.

[14] See Bonroy and Constantatos (2008) on the effects of perfect labels on equilibrium.

Proposition 4 *Under perfect information, when the conditions of lemma 2 hold, both firms are active at equilibrium if and only if:*

$$\theta_{\max} - 2\theta_{\min} > \Delta u.$$

Otherwise only the firm selling the high-quality good is active in the market.

Indeed, in the case $2\Delta u > \frac{\theta_{\max}-\theta_{\min}}{\overline{m}-\underline{m}}$[15] as stated in lemma 2, an interior solution can only arise if consumers' preferences with respect to the horizontal characteristic are biased towards good 2, i.e. $\theta_{\max} - 2\theta_{\min} > \Delta u$. If this condition is not satisfied, the equilibrium price of good 2 would be negative. Therefore, there is only space for the high-quality good in this industry. At equilibrium, the monopolist (firm 1) would charge a price p_1^{*PI} equal to

$$p_1^{*PI} = \theta_{\min} + \Delta u,$$

which prevents entry of firm 2 even when $p_2^{*PI} = 0$ (limit price).

Accordingly, there are situations in which the low quality firm can survive only due to consumers' imperfect information. Informing consumers would lead to the exclusion of the low quality firm, thus entailing a more concentrated equilibrium market structure.[16] In spite of this, it can be shown that the loss of profits by the low quality firm is more than compensated by the welfare improvement for the other agents.

Another benchmark to compare our solutions would consist in comparing the equilibria obtained in our analysis with the one corresponding to a fully deterministic vertical differentiation model with variants' qualities defined by

$$\begin{aligned} u_1 &= \frac{\underline{m}+\overline{m}}{2} u_h + \left(1 - \frac{\underline{m}+\overline{m}}{2}\right) u_l \\ u_2 &= \frac{\underline{m}+\overline{m}}{2} u_l + \left(1 - \frac{\underline{m}+\overline{m}}{2}\right) u_h. \end{aligned}$$

This corresponds to replace the uncertainty bearing on the identity of the firms by the certainty equivalent obtained by the average consumer, say consumer $\hat{m}$, whose beliefs coincide with the average belief[17] $\hat{m} = \frac{\underline{m}+\overline{m}}{2}$.

Notice that, by focusing on consumer $\hat{m}$, we neutralize consumers' dispersion of beliefs since all consumers are now assumed to have the same beliefs as consumer $\hat{m}$. Consequently, as in the case of perfect information, we end up with a pure horizontal differentiation model, with consumers' heterogeneity bearing exclusively on the interval $[\theta_{\min}, \theta_{\max}]$. In this case, when both firms are active at equilibrium, prices are equal to:

$$\begin{aligned} p_1^{H*} &= \tfrac{1}{3}\left(2\theta_{\max} - \theta_{\min} + (2\hat{m}-1)\Delta u\right), \\ p_2^{H*} &= \tfrac{1}{3}\left(\theta_{\max} - 2\theta_{\min} - (2\hat{m}-1)\Delta u\right), \end{aligned}$$

[15] In the case of perfect information ($\underline{m} = \overline{m} = 1$), the vertical dominance as stated in lemma 2 establishes that vertical differentiation must be sufficiently strong: $\Delta u > \frac{\theta_{\max}-3\theta_{\min}}{2}$.

[16] It is worth noting that the result in the previous Porposition is, in part driven, by the assumption that the costs of production are the same for both firms.

[17] In particular, when $\hat{m} = 1$, the case of certainty equivalent coincides with the case of perfect information when good 1 is the high-quality good.

which coincide with the prices obtained when $2\Delta u < \frac{\theta_{\max}-\theta_{\min}}{\overline{m}-\underline{m}}$,with beliefs uniformly distributed on the interval $[\underline{m}, \overline{m}]$ (see (6)).

4 An application to the newspapers´ industry

In this section, we illustrate how the abstract analysis proposed above can be applied to a specific industry. In order to keep in line with one of the examples given above, this section investigates competition in the newspapers' industry. We consider two newspapers or weekly magazines representing the ideas of two different political parties: one left wing newspaper (say newspaper 1) and one right wing newspaper (newspaper 2). The political spectrum of the readers is represented by the interval $[\theta_{\min}, \theta_{\max}]$, in which readers are uniformly distributed, each point in the interval representing simultaneously a specific political opinion and the reader for whom this opinion is its *"ideal"* one. For instance, $\theta_{\min} < 0$ corresponds to the reader who has the most extreme left opinion and $\theta_{\max} > 0$ the most extreme right one. The political opinion dimension clearly corresponds to the horizontal component of readers' preferences, given by (1) and (2). A reader of type $\theta \in [\theta_{\min}, \theta_{\max}]$ values the political ideas expressed in newspaper 1 and dislikes the political views of newspaper 2 when $\theta > 0$. The reverse situation happens if the readers' ideal political opinion satisfies $\theta < 0$.

However, readers' preferences over the two newspapers not only depend on the political opinion they represent but also on the degree of accuracy with which news are selected and dispatched by each of the newspapers. From that viewpoint, all readers agree that the higher this accuracy, the better. This represents the vertical dimension of readers' preferences in (1) and (2). To be more precise, in the context of this application to the newspaper industry, u_h denotes utility impact of the most accurate newspaper, whereas u_l corresponds to the utility impact of the least accurate newspaper. We denote by Δu, $\Delta u = u_h - u_l$, the resulting "quality gap". Of course, readers do not know with certainty which newspaper has the higher accuracy. Furthermore, readers' beliefs regarding the newspapers' credibility are heterogeneous. In the context of our model, the trust of a particular reader in the accuracy of news in each newspaper is represented by the number m, which corresponds to the subjective probability that the reader of type m assigns to the event

$$\{\textit{newspaper 1 corresponds to the most credible newspaper}\}.$$

We assume that m is uniformly distributed in the "domain of beliefs" $[\underline{m}, \overline{m}]$, with $0 \leq \underline{m} < \frac{1}{2} < \overline{m} \leq 1$.

As shown in Proposition 1 and Proposition 2 above, the prices quoted by newspapers in equilibrium depend on the compliance (or not) of condition (5). When the dispersion of political opinions is more significant than the dispersion of beliefs regarding the newspaper's credibility, condition (5) holds. Thus, under the conditions in Lemma 1, equilibrium prices in the newspaper market are given by (6) and they depend on two factors: *(i)* the popularity of each newspaper's political viewpoint within the universe of readers;[18] and *(ii)* the average beliefs about the credibility of each newspaper. When the dispersion of political opinions is less significant than the dispersion of beliefs regarding the newspaper's credibility, condition (5) is violated. Under the conditions in Lemma 2, the equilibrium prices are (7), which depend on two factors: *(i)* the average political opinion among the universe of readers; and *(ii)* the readers' beliefs about the credibility of each newspaper.[19]

In light of the previous results, we now study how a shock in readers' beliefs may affect the nature of price competition between firms. To this end, we consider *(i)* a spectrum of political opinions given by $[\theta_{\min}, \theta_{\max}] = \left[-\frac{1}{4}, \frac{3}{4}\right]$, with the political viewpoint of newspaper 1 being more popular among the population of readers; and *(ii)* a domain of beliefs equal to $\left[\frac{1}{4}, \frac{3}{4}\right]$, which both newspapers believed to be equally credible among the population of readers. Then we assume, for example, that it is publicly revealed that newspaper 1 has hidden important news, while newspaper 2 has made reference to them. As a result, all readers are led to revise their beliefs about the trust they have in each newspaper. Of course, while beliefs were symmetric before this exogenous shock, now all readers become more suspicious about newspaper 1 so that the domain of beliefs switches from $\left[\frac{1}{4}, \frac{3}{4}\right]$ to, say, $\left[\frac{1}{10}, \frac{6}{10}\right]$.

Starting with a "quality gap" Δu equal to 0.2, we have that, before the shock in the newspaper's 1 credibility, condition (5) is satisfied and equilibrium prices are given by $p_1^{H*} = 0.58$ and $p_2^{H*} = 0.42$, since both the conditions in Lemma 1 hold for the values of the parameters. After the shock, we have that, everything else the same, the condition (5), as well as the conditions in Lemma 1 remain valid, with equilibrium prices being now equal to $\widetilde{p}_1^{H*} = 0.56$ and $\widetilde{p}_2^{H*} = 0.44$. As expected, we obtain that the equilibrium price of the newspaper which has now become less credible decreases, whereas the price of the rival newspaper increases. Yet, newspaper 1 remains more expensive since its political viewpoint remains the most popular one.

However, the revelation of the behavior of a newspaper in hiding information may affect preferences in more than one way. For example, the revelation of this kind of behavior on the part of newspapers may cause a "psychological shock" among the readers' population, so that readers now start to pay much more attention to the accuracy of news. As a result, the impact of news' accuracy on readers' preferences changes. For example, if the "quality gap" between the two newspapers expands from $\Delta u = \frac{2}{10}$ to $\Delta u = 2$, condition (5) is no longer satisfied. As a result, the equilibrium prices quoted by newspapers are no longer given by (6). Instead, equilibrium prices are equal to (7) since the conditions in Lemma 2 are satisfied for the selected values of the parameters. Accordingly, at the new equilibrium after the shock, we have $p_1^{V*} = 0.97$ and $p_2^{V*} = 1.03$.

[18] The popularity of the newspapers' political viewpoints depend on the support of the distribution of θ, given by $[\theta_{\min}, \theta_{\max}]$. If $\theta_{\max} > -\theta_{\min}$, the political viewpoint of newspaper 1 is more popular among readers (which, everything else the same, makes newspaper 1 more expensive than newspaper 2). On the contrary, if $\theta_{\max} < -\theta_{\min}$, the political viewpoint of newspaper 2 is the dominant one among the universe of readers.

[19] These beliefs depend on the domain of beliefs $[\underline{m}, \overline{m}]$

Comparing these prices with $\widetilde{p}_1^{H*} = 0.56$ and $\widetilde{p}_2^{H*} = 0.44$, we observe that the expansion in the value of Δu allows both newspapers to increase their prices (including the now less credible newspaper). However, after the shock in Δu, the now more credible newspaper becomes more expensive. Despite being the newspaper conveying the least popular political viewpoint, after the expansion of Δu, the dispersion of beliefs becomes more significant than the dispersion of political opinions. This leads the more credible (yet less popular newspaper) to charge a higher price than its rival.

5 Conclusion

In this paper, we have considered price competition when variants are defined along two dimensions (horizontal and vertical) and consumers are uncertain about which firm sells which quality. We focus on the case of credence goods, in which the quality of the good can be hardly ascertained by simple experimentation.

The paper identifies sufficient conditions for existence and uniqueness. These conditions correspond in a wide domain of the parameters. Interestingly, this domain comprises two distinct sub-domains, according as the quality gap between the goods is sufficiently small (resp. large) when compared to the ratio between the dispersion of consumers' tastes and the dispersion of consumers' beliefs about quality. In each sub-domain we characterize the equilibrium prices with respect to the main magnitudes of the model: *(i)* size of the quality gap, *(ii)* dispersion of tastes; and *(iii)* dispersion of beliefs.

Then, we apply our model assuming perfect information, in which consumers know which firm sells the high quality good. Perfect information, by eliminating the dispersion of consumers' beliefs engenders a pure horizontally differentiated model. We conclude that, when the quality gap is sufficiently large and consumers are perfectly informed about it, informing consumers could lead to the exclusion of the low quality firm (while, it could survive in a setting of imperfect information).

We have also considered the certainty equivalent outcome, in which the dispersion of consumers' beliefs is eliminated and replaced by the beliefs of the average consumer. In this benchmark case, equilibrium prices coincide with those obtained in a setting of imperfect information when the dispersion of tastes is more significant than the dispersion of beliefs. Moreover, when the average consumer is perfectly informed, the equilibrium prices corresponding to the certainty equivalent outcome coincide with those obtained under perfect information.

Finally, we have shown how to apply the model to the case of a specific industry, the newspaper's market. Newspapers are credence goods when horizontal preferences for political opinions are combined with the degree of accuracy in news provision attached by readers to each newspaper. The analysis allows us to show how an external shock influencing both the domain of the beliefs of readers and their relative valuation of the newspapers' credibility affects the equilibrium prices.

Applying the same abstract model to other industries in which credence goods are produced and exchanged would be a fruitful field for future research. We have already quoted the market for restaurants. It could also be applied to the industry of the electromenager, when the washing machine may break down, or to the industry of car repairs,when the car is about to die. More generally, it could apply to all situations when commissioning an expert would be necessary to ascertain the quality of a product, without even being sure that it would be ascertained after the visit of the expert...

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Appendix

Appendix A: The demand functions

According to (4), at a given pair of prices (p_1, p_2), the set of consumers can be partitioned into two subsets, each describing those consumers who buy good 1 and good 2, respectively. For a given value of $m \in [\underline{m}, \overline{m}]$, the subset of consumers buying good 1 is given by $\left[\tilde{\theta}(m), \theta_{\max}\right]$, while the subset of consumers buying good 2 is given by $\left[\theta_{\min}, \tilde{\theta}(m)\right]$.

The following figures illustrate the structure of demands for different pairs of prices (p_1, p_2). The first figures represent the various configurations of demand when $2\Delta u < \frac{\theta_{\max} - \theta_{\min}}{\overline{m} - \underline{m}}$.

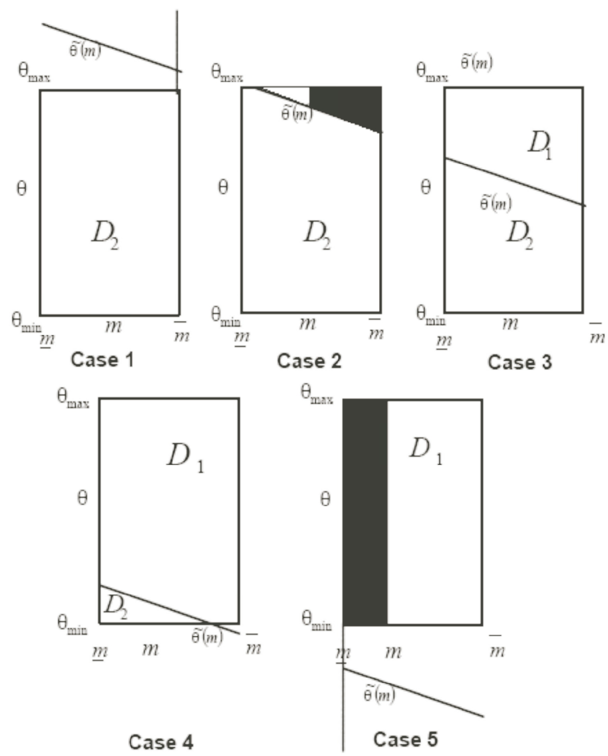

Fig.3 - Demands' configuration when $2\Delta u < \dfrac{\theta_{\max} - \theta_{\min}}{\overline{m} - \underline{m}}$

Each rectangle in the figure above identifies a specific partition of the set of consumers corresponding to different pairs of prices (p_1, p_2). Case 1 corresponds to a value of p_1 which, given p_2, is so high that there is no consumer willing to buy variant 1 at that price. In cases 2, 3 and 4, both firms are active in the market. In case 2, the demand of firm 1 corresponds to the area of the shadow triangle. When p_1 further decreases, we move to case 3, where all types m are served by both firms and the demand of firm 1 now consists of an area which is the sum of a triangle and a rectangle. When p_1 even further decreases, we move to case 4 and now demand corresponds to an area which is the sum of a triangle and two rectangles. Finally, in case 5, firm 1 becomes a monopolist and the demand of firm 2 is equal to zero.

When $2\Delta u > \frac{\theta_{\max}-\theta_{\min}}{\overline{m}-\underline{m}}$, similar comments apply *mutatis mutandis* and the structure of demand is represented in the following figure.

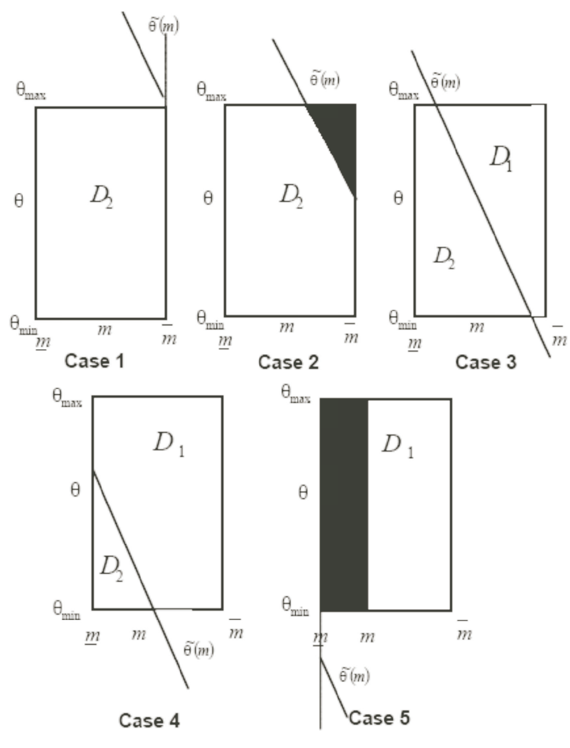

Fig.4 - Demands' configuration when $2\Delta u < \dfrac{\theta_{\max}-\theta_{\min}}{\overline{m}-\underline{m}}$

Addressing first the case when $2\Delta u < \frac{\theta_{\max}-\theta_{\min}}{\overline{m}-\underline{m}}$, we define by R_i^H the set $R_i^H = \{(p_1, p_2) : (p_1, p_2)$ *leads to a demand corresponding to case* $i, i = 1, ..., 5\}$.

In this setting, we obtain the analytical expressions of firms' demands as:

$$D_1^H(p_1,p_2)=\begin{cases} 0 & if \quad (p_1,p_2)\in R_1^H \\ \frac{1}{4}\frac{(\theta_{\max}-p_1+p_2+(2\overline{m}-1)\Delta u)^2}{(\overline{m}-\underline{m})(\theta_{\max}-\theta_{\min})\Delta u} & if \quad (p_1,p_2)\in R_2^H \\ \frac{\theta_{\max}-p_1+p_2+(\underline{m}+\overline{m}-1)\Delta u}{\theta_{\max}-\theta_{\min}} & if \quad (p_1,p_2)\in R_3^H \\ 1-\frac{1}{4}\frac{(\theta_{\min}-p_1+p_2-(1-2\underline{m})\Delta u)^2}{(\overline{m}-\underline{m})(\theta_{\max}-\theta_{\min})\Delta u} & if \quad (p_1,p_2)\in R_4^H \\ 1 & if \quad (p_1,p_2)\in R_5^H, \end{cases}$$

and

$$D_2^H(p_1,p_2)=1-D_1^H(p_1,p_2),$$

Observing figure 3, it becomes evident that R_I^H is observed whenever $\theta(\overline{m})>\theta_{\max}$, or equivalently:

$$R_1^H=\{(p_1,p_2):p_1-p_2>\theta_{\max}+(2\overline{m}-1)\Delta u\}.$$

From figure 3, it follows as well that R_2^H is observed whenever: $\theta(\underline{m})>\theta_{\max}$ and, simultaneously $\theta_{\min}<\theta(\overline{m})<\theta_{\max}$. When $2\Delta u<\frac{\theta_{\max}-\theta_{\min}}{\overline{m}-\underline{m}}$, these conditions imply:

$$R_2^H=\{(p_1,p_2):(2\underline{m}-1)\Delta u+\theta_{\max}<p_1-p_2<\theta_{\max}+(2\overline{m}-1)\Delta u\}.$$

Similarly, R_3^H is observed when $\theta(\overline{m})>\theta_{\min}$ and $\theta(\underline{m})<\theta_{\max}$, which, under $2\Delta u<\frac{\theta_{\max}-\theta_{\min}}{\overline{m}-\underline{m}}$, are equivalent to:

$$R_3^H=\{(p_1,p_2):\theta_{\min}+(2\overline{m}-1)\Delta u<p_1-p_2<\theta_{\max}+(2\underline{m}-1)\Delta u\}.$$

R_4^H is observed when $\theta_{\min}<\theta(\underline{m})<\theta_{\max}$ and, simultaneously, $\theta(\overline{m})<\theta_{\min}$. This is equivalent to

$$R_4^H=\{(p_1,p_2):\theta_{\min}+(2\underline{m}-1)\Delta u<p_1-p_2<\theta_{\min}+(2\overline{m}-1)\Delta u\}.$$

Finally, under R_5^H firm 1 is a monopolist, which is observed whenever $\theta(\underline{m})<\theta_{\min}$, or equivalently:

$$R_5^H=\{(p_1,p_2):p_1-p_2<\theta_{\min}+(2\underline{m}-1)\Delta u\}.$$

When $2\Delta u<\frac{\theta_{\max}-\theta_{\min}}{\overline{m}-\underline{m}}$, we define by R_i^V the set $R_i^V=\{(p_1,p_2):(p_1,p_2)$ *leads to a demand corresponding to case* $i, i=1,...,5\}$. In this setting, the analytical expressions of firms' demands are given by:

$$D_1^V(p_1,p_2)=\begin{cases} 0 & if \quad (p_1,p_2)\in R_1^V \\ \frac{1}{4}\frac{(\theta_{\max}-p_1+p_2+(2\overline{m}-1)\Delta u)^2}{(\overline{m}-\underline{m})(\theta_{\max}-\theta_{\min})\Delta u} & if \quad (p_1,p_2)\in R_2^V \\ \frac{1}{4}\frac{\theta_{\max}+\theta_{\min}-2p_1+2p_2-2(1-2\overline{m})\Delta u}{\Delta u(\overline{m}-\underline{m})} & if \quad (p_1,p_2)\in R_3^V \\ 1-\frac{1}{4}\frac{(\theta_{\min}-p_1+p_2-(1-2\underline{m})\Delta u)^2}{(\overline{m}-\underline{m})(\theta_{\max}-\theta_{\min})\Delta u} & if \quad (p_1,p_2)\in R_4^V \\ 1 & if \quad (p_1,p_2)\in R_5^V, \end{cases}$$

and

$$D_2^V(p_1, p_2) = 1 - D_1^V(p_1, p_2),$$

with each demand region being given by:

$$\begin{aligned}
R_1^V &= \{(p_1, p_2) : p_1 - p_2 > \theta_{\max} + (2\overline{m} - 1)\Delta u\}, \\
R_2^V &= \{(p_1, p_2) : \theta_{\min} + (2\overline{m} - 1)\Delta u < p_1 - p_2 < \theta_{\max} + (2\overline{m} - 1)\Delta u\}, \\
R_3^V &= \{(p_1, p_2) : \theta_{\max} + (2\underline{m} - 1)\Delta u < p_1 - p_2 < \theta_{\min} + (2\overline{m} - 1)\Delta u\}, \\
R_4^V &= \{(p_1, p_2) : \theta_{\min} + (2\underline{m} - 1)\Delta u < p_1 - p_2 < \theta_{\max} + (2\underline{m} - 1)\Delta u\}, \\
R_5^V &= \{(p_1, p_2) : p_1 - p_2 < \theta_{\min} + (2\underline{m} - 1)\Delta u\}.
\end{aligned}$$

Appendix B - Proofs

Lemma 5 ***Sufficient conditions when*** $2\Delta u < \frac{\theta_{\max}-\theta_{\min}}{\overline{m}-\underline{m}}$
The condition

$$\theta_{\max} \geq (1+\overline{m}-3\underline{m})\,\Delta u \tag{10}$$

is sufficient to guarantee that profits of firm 1 *are decreasing in* p_1 *when* $(p_1,p_2)\in R_2^H$*. Similarly profits of firm 2 are decreasing in* p_2 *when* $(p_1,p_2)\in R_4^H$ *if*

$$-\theta_{\min} \geq (3\overline{m}-\underline{m}-1)\,\Delta u \tag{11}$$

Proof. In $(p_1,p_2)\in R_2^H$, firm $1's$ profits are given by a third-degree polynomial:

$$\pi_1(p_1,p_2)_{\rceil R_2^H} = \left(\frac{1}{4}\frac{(\theta_{\max}-p_1+p_2+(2\overline{m}-1)(\Delta u))^2}{(\overline{m}-\underline{m})(\theta_{\max}-\theta_{\min})(\Delta u)}\right)p_1,$$

with $\lim_{p_1\to\infty}\left(\left(\frac{1}{4}\frac{(\theta_{\max}-p_1+p_2+(2\overline{m}-1)(\Delta u))^2}{(\overline{m}-\underline{m})(\theta_{\max}-\theta_{\min})(\Delta u)}\right)p_1\right)=+\infty.$ ■

The extremes of $\pi_1(p_1,p_2)_{\rceil R_2^H}$ are obtained as $\frac{\partial\left(\pi_1(p_1,p_2)_{\rceil R_2^H}\right)}{\partial p_1}=0$, yielding:

$$\hat{p}_1(p_2) = \theta_{\max}+p_2+(2\overline{m}-1)(\Delta u) \tag{12}$$

$$\check{p}_1(p_2) = \frac{1}{3}(\theta_{\max}+p_2+(2\overline{m}-1)(\Delta u)) \tag{13}$$

Notice that, for a given p_2, the price level $\hat{p}_1(p_2)$ corresponds to the switching price between R_2^H and R_I^H (where firm 1 is evicted from the market, obtaining nil profits). Accordingly, firm 1 will never have any incentives to make an unilateral deviation towards $\hat{p}_1(p_2)$.

Furthermore, it is easy to see that:

$$\frac{\partial\pi_1(p_1,p_2)_{\rceil R_2^H}}{\partial p_1}\rceil^{+}_{\check{p}_1(p_2)} < 0 \tag{14}$$

$$\frac{\partial\pi_1(p_1,p_2)_{\rceil R_2^H}}{\partial p_1}\rceil^{-}_{\check{p}_1(p_2)} > 0. \tag{15}$$

Accordingly, if $\check{p}_1(p_2)$ occurs for $(p_1,p_2):\check{p}_1(p_2)-p_2<(2\underline{m}-1)(\Delta u)+\theta_{\max}$, we observe that $\check{p}_1(p_2)$ would be outside (at the left) of R_2^H. Thus, considering (12)-(15), for any $(p_1,p_2)\in R_2^H$, we observe that $\frac{\partial\pi_1(p_1,p_2)_{\rceil R_2^H}}{\partial p_1}<0$, preventing any incentives for advantageous unilateral deviations by firm 1. To end the proof, plug (13) in the condition

$$\check{p}_1(p_2)-p_2<(2\underline{m}-1)(\Delta u)+\theta_{\max}, \tag{16}$$

obtaining:

$$p_2 > -(\theta_{\max}+(3\underline{m}-\overline{m}-1)(\Delta u)) \tag{17}$$

Given that prices must be non-negative, a sufficient condition to guarantee that (17) holds is:

$$-\left(\theta_{\max}+\left(3\underline{m}-\overline{m}-1\right)(\Delta u)\right)<0 \Leftrightarrow$$
$$\Leftrightarrow \theta_{\max}>\left(1+\overline{m}-3\underline{m}\right)(\Delta u),$$

which corresponds to condition (10) in Lemma 1.

Concerning the second condition in lemma 2, one must analyze profits of firm 2 under the case of horizontal dominance. In $(p_1,p_2)\in R_4^H$, firm $2's$ profits are given by a third-degree polynomial:

$$\pi_2\left(p_1,p_2\right)_{\rceil R_4^H}=\left(\frac{1}{4}\frac{\left(\theta_{\min}-p_1+p_2-\left(1-2\underline{m}\right)(\Delta u)\right)^2}{\left(\overline{m}-\underline{m}\right)\left(\theta_{\max}-\theta_{\min}\right)(\Delta u)}\right)p_2,$$

with $\lim_{p_2\to\infty}\left(\left(\frac{1}{4}\frac{\left(\theta_{\min}-p_1+p_2-\left(1-2\underline{m}\right)(\Delta u)\right)^2}{\left(\overline{m}-\underline{m}\right)\left(\theta_{\max}-\theta_{\min}\right)(\Delta u)}\right)p_2\right)=+\infty.$

The extremes of $\pi_2\left(p_1,p_2\right)_{\rceil R_4^H}$ are obtained as $\frac{\partial\left(\pi_2\left(p_1,p_2\right)_{\rceil R_4^H}\right)}{\partial p_2}=0$, yielding:

$$\hat{p}_2\left(p_1\right) = -\theta_{\min}+p_1-\left(2\underline{m}-1\right)(\Delta u) \tag{18}$$
$$\check{p}_2\left(p_1\right) = \frac{1}{3}\left(-\theta_{\min}+p_1-\left(2\underline{m}-1\right)(\Delta u)\right) \tag{19}$$

Notice that, for a given p_1, the price level $\hat{p}_2\left(p_1\right)$ corresponds to the switching price between R_4^H and R_5^H (where firm 2 is evicted from the market, obtaining nil profits). Accordingly, firm 2 will never have incentives to make an unilateral deviation towards $\hat{p}_2\left(p_1\right)$.

Furthermore, it is easy to see that

$$\frac{\partial\pi_2\left(p_1,p_2\right)_{\rceil R_4^H}}{\partial p_2}\rceil_{\check{p}_2(p_1)}^{-} > 0 \tag{20}$$
$$\frac{\partial\pi_2\left(p_1,p_2\right)_{\rceil R_4^H}}{\partial p_2}\rceil_{\check{p}_2(p1)}^{+} < 0. \tag{21}$$

Accordingly, if $\check{p}_2\left(p_1\right)$ occurs for $(p_1,p_2): p_1-\check{p}_2\left(p_1\right)>\theta_{\min}+\left(2\overline{m}-1\right)(\Delta u)$, we observe that $\check{p}_2\left(p_1\right)$ would be outside (at the left) of R_4^H . Thus, considering (20)-(21), for any $(p_1,p_2)\in R_4^H$, we observe that $\frac{\partial\pi_2(p_1,p_2)_{\rceil R_4^H}}{\partial p_2}<0$, preventing any incentives for advantageous unilateral deviations by firm 1. To end the proof, plug (19) in the condition

$$p_1-\check{p}_2\left(p_1\right)>\theta_{\min}+\left(2\overline{m}-1\right)(\Delta u), \tag{22}$$

obtaining

$$p_1>\theta_{\min}-\left(\underline{m}-3\overline{m}+1\right)(\Delta u) \tag{23}$$

Given that prices must be non-negative, a sufficient condition to guarantee that the previous condition holds is:

$$\theta_{\min} - (\underline{m} - 3\overline{m} + 1)(\Delta u) < 0 \Leftrightarrow$$
$$\Leftrightarrow -\theta_{\min} > (3\overline{m} - 1 - \underline{m})(\Delta u),$$

which corresponds to condition (11) in Lemma 1.■

Lemma 6 ***Sufficient conditions when*** $2\Delta u > \frac{\theta_{\max}-\theta_{\min}}{\overline{m}-\underline{m}}$
The condition

$$\Delta u > \frac{\theta_{\max} - 3\theta_{\min}}{2(2\overline{m} - 1)} \tag{24}$$

is sufficient to guarantee that profits of firm 1 *are decreasing in* p_1 *when* $(p_1, p_2) \in R_2^V$. *Similarly profits of firm* 2 *are decreasing in* p_2 *when* $(p_1, p_2) \in R_4^V$ *if*

$$\Delta u > \frac{3\theta_{\max} - \theta_{\min}}{2(1 - 2\underline{m})}.$$

Proof. Notice that all the comments concerning the profits of firm 1 up to expression (15) in the proof of lemma 1 also hold in the case of vertical dominance. However, considering the price domains, in the case of vertical dominance, condition (16) becomes

$$\check{p}_1(p_2) - p_2 < \theta_{\min} + (2\overline{m} - 1)(\Delta u),$$

which is equivalent to:

$$p_2 > \frac{1}{2}(\theta_{\max} - 3\theta_{\min} - 2(2\overline{m} - 1)(\Delta u)).$$

■

Given non-negativity of prices a sufficient condition to guarantee that the above inequality holds for all feasible p_2 is:

$$\frac{1}{2}(\theta_{\max} - 3\theta_{\min} - 2(2\overline{m} - 1)(\Delta u)) < 0 \Leftrightarrow$$
$$\Leftrightarrow \Delta u > \frac{\theta_{\max} - 3\theta_{\min}}{2(2\overline{m} - 1)},$$

which corresponds to the first condition in lemma 2.

Similarly, all the comments concerning the profits of firm 2 until expression (21) also hold in the case of vertical dominance. However, considering the price domains, in the case of vertical dominance, condition (22) becomes

$$p_1 - \check{p}_2(p_1) > \theta_{\max} + (2\underline{m} - 1)(\Delta u),$$

which is equivalent to:

$$p_1 > \frac{1}{2}\left(3\theta_{\max} - \theta_{\min} + 2\left(2\underline{m} - 1\right)(\Delta u)\right).$$

Given non-negativity of prices a sufficient condition to guarantee that the above inequality holds for all feasible p_2 is:

$$\frac{1}{2}\left(3\theta_{\max} - \theta_{\min} + 2\left(2\underline{m} - 1\right)(\Delta u)\right) < 0 \Leftrightarrow$$
$$\Leftrightarrow \Delta u > \frac{3\theta_{\max} - \theta_{\min}}{2\left(1 - 2\underline{m}\right)},$$

which corresponds to the second condition in lemma 2.■

Proof of Proposition 1

When the conditions in lemma 1 hold, firms' profits are quasi-concave functions of prices and they reach only one maximum value in the region R_3^H where the profit functions write as

$$\pi_1^H\left(p_1, p_2\right) = \left(\frac{\theta_{\max} - p_1 + p_2 + \left(\underline{m} + \overline{m} - 1\right)\Delta u}{\theta_{\max} - \theta_{\min}}\right) p_1,$$

for firm 1, and

$$\pi_2^H\left(p_1, p_2\right) = \left(1 - \frac{\theta_{\max} - p_1 + p_2 + \left(\underline{m} + \overline{m} - 1\right)\Delta u}{\theta_{\max} - \theta_{\min}}\right) p_2,$$

for firm 2.

In light of the properties of the profit functions, the profit-maximizing prices are obtained by the following system of equations:

$$\begin{cases} \frac{\partial \pi_1^H(p_1,p_2)}{\partial p_1}\rceil_{R_3^H} = 0 \\ \frac{\partial \pi_2^H(p_1,p_2)}{\partial p_2}\rceil_{R_3^H} = 0 \end{cases},$$

leading to the equilibrium prices presented in Proposition 1. ■

Proof of Proposition 2

When the conditions in lemma 2 hold, firms' profits are quasi-concave functions of prices and they reach only one maximum value in the region R_3^V where the profit functions write as

$$\pi_1^V\left(p_1, p_2\right) = \left(\frac{1}{4}\frac{\theta_{\max} + \theta_{\min} - 2p_1 + 2p_2 - 2\left(1 - 2\overline{m}\right)\Delta u}{\Delta u\left(\overline{m} - \underline{m}\right)}\right) p_1,$$

for firm 1, and

$$\pi_2^V(p_1,p_2) = \left(1 - \frac{1}{4}\frac{\theta_{\max} + \theta_{\min} - 2p_1 + 2p_2 - 2\left(1 - 2\overline{m}\right)\Delta u}{\Delta u\left(\overline{m} - \underline{m}\right)}\right) p_2,$$

for firm 2.

In light of the properties of the profit functions, the profit-maximizing prices are obtained by the following system of equations:

$$\begin{cases} \frac{\partial \pi_1^V(p_1,p_2)}{\partial p_1} \rceil_{R_3^V} = 0 \\ \frac{\partial \pi_2^V(p_1,p_2)}{\partial p_2} \rceil_{R_3^V} = 0 \end{cases},$$

leading to the equilibrium prices presented in Proposition 2. ■

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