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Existence and multiplicity of temporary equilibria
under nominal price rigidities

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CORE

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Abstract

This paper proves existence of a multiplicity of equilibria in a 2-period TGE model with money and nominal rigidities. Households have incomplete preferences and firms reach decisions through the “control principle” (cf. Dreze 1989).

Keywords: temporary general equilibrium, incomplete preferences, price rigidities

JEL Classification: D51, D52

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1 Introduction

1.1. This paper proves a result, stated informally as Theorem 5.1 in my (2009) Borch Lecture, concerning existence of a multiplicity of equilibria, in a 2-period Temporary General Equilibrium (TGE) model with money and (downward) nominal price rigidities.

Relative to the canonical TGE model, one innovation concerns the introduction of a convincing (to me!) decision criterion for firms, namely the “control principle” introduced in Drèze (1989) for the GEI model. The resulting criterion is defined in Section 3. That criterion generates a partial ordering of production plans and existence of equilibrium is proved by invoking the Shafer-Sonnenschein (1975) theorem.

In this paper, I proceed similarly from incomplete household preferences. This is an important step towards realism, in particular regarding portfolio choices. As noted in Section 2.2, the standard approach to TGE assumes that households assign subjective probability densities to future signals (prices, quantity constraints and dividends). I proceed here without that extreme¹ assumption and rely instead on weakenings of the standard assumptions of convexity and continuity (Section 2.1). These weakenings (borrowed from Shafer-Sonnenschein) are of independent interest.

Another (minor, but relevant) innovation concerns the modeling of money, which allows here for different interest rates on bankloans and bonds. Otherwise, the model and assumptions are standard.

¹I for one do not satisfy it – so, under Muthian rational expectations....

1.2. The economy extends over 2 periods with S possible states tomorrow $s = 1, \dots, S$ and state 0 today. I write $\bar{S}$ for the set $\bar{S} = \{0, 1, \dots, S\}$ and for its cardinality $S + 1$, and I write S for the set $S = \{1, \dots, S\}$. There are L commodities, $\ell = 1, \dots, L$, with $L = L^I \cup L^{II}$ and with prices $(p_0, p_1, \dots, p_S) := (p^I, p^{II})$ in $R_+^{L(S+1)}$; for $\ell \in L^{II}$, $p_{\ell 0} \geq \tilde{p}_{\ell 0} > 0$ (downward nominal rigidities). Commodity 1 belongs to L^I .

There are H households, $h = 1, \dots, H$, J firms, $j = 1, \dots, J$, and a (central) bank with index $j = 0$. Initial ownership fractions are $\bar{\theta}^{hj} \geq 0$, $\sum_h \bar{\theta}^{hj} = 1$ for all $j = 0, 1, \dots, J$.² There is a stock market at time 0 with market prices for shares π in R_+^J . The profits of firm j in state s are denoted V_s^j , $j = 1, \dots, J$, $s = 0, 1, \dots, S$. The profits of the bank are similarly denoted V_s^0 .

Prices and profits are evaluated as of beginning of periods.

Transactions take place in continuous time and consist in exchange of commodities (or services) for “cash”. Households and firms keep accounts with the bank; deposits (positive balances) earn no interest; they are thus fully equivalent to (here stand for) cash holdings; overdrafts (negative balances) are subject to interest charges at rates $r = (r_0, r_1, \dots, r_S)$, exogenously given (set by the bank); interest is paid at end of period.

There is a safe nominal bond issued at the beginning of period 0 for a price $\varphi \in [0, 1]$ and paying 1 at time 1 (end of period 0) in every state. Define $\rho_0 := \frac{1-\varphi}{\varphi}$, so that $\varphi = \frac{1}{1+\rho_0}$; $\rho_0 \geq 0$ is the interest rate on bonds – a market clearing rate.

²Thus, the (central) bank is owned by households. For simplicity, and analogy with public ownership, bank shares are not traded.

The bond transactions of a household b^h may have either one of two purposes: (i) to reallocate resources between periods 0 and 1; or (ii) to anticipate financial flows linked to current trades in commodities. The second purpose may consist in buying bonds paid for by a bank loan (overdraft) to be reimbursed by the proceeds of net sales occurring early in the period; or in selling bonds and holding the proceeds in (cash or) bank deposit to finance net purchases early in the period.

From the viewpoint of the households, all the type (ii) financial transactions are in the nature of interest swaps; they leave unchanged the gross proceeds from trade in commodities; they enter the budget constraint by adding interest terms to the trade proceeds – but do not otherwise affect preferences or feasibility.

From the viewpoint of the bank, the beginning of period balance sheet consists of deposits and loans, as resulting from the financial transactions of households just described. As the period unfolds, the households deposit the proceeds of their sales or withdraw the value of their purchases. In the aggregate, these amounts match, so that the balance sheet of the bank remains constant.

It simplifies notation w.l.o.g. to assume that the bond transactions of households amounting to interest swaps are met by the bank and settled at the end of the initial period – thus do not enter the period 1 budgets. The initial balance sheet of the bank will thus include bond holdings when deposits exceed overdrafts, or bond sales when overdrafts exceed deposits. As for households, the bonds settled (with the bank) at the end of period 0 enter that period's budget constraints for interests only.

In period 1, state s , ρ_s is an interest rate on time deposits, set by the bank.

2 Households

2.1. Households consume vectors $x^h \in X^h \subset R^{\bar{S}L}$; they are endowed with initial resources $\omega^h \in \text{int } X^h$; their net trades are denoted $z^h := x^h - \omega^h$. To satisfy a cash-in-advance constraint in continuous time, they may hold initial bank deposits $m_{ds}^h \geq 0$, $s \in \bar{S}$, financed by bonds³ repaid with interest $\rho_s m_{ds}^h$ at end of period. I treat the timing of transactions (net trades) within periods as exogenous; accordingly, net trades z_s^h at prices p_s with initial deposit m_{ds}^h may entail bank account overdrafts of average (over time) level $\bar{m}_s^h$ entailing interest costs $r_s \bar{m}_s^h$. The transactions technology defines, conditionally on p_s , the set of feasible triplets $(z_s^h, m_{ds}^h, \bar{m}_s^h \mid p_s)$, $s \in \bar{S}$; I denote that set by $\mathcal{X}_s^h(p_s) \subset R^L \times R_+^2$. I also write $m_s^h := (m_{ds}^h, \bar{m}_s^h)$. And I define

$$\tilde{X}^h(p) = \{(x, m) \mid x \in X^h, ((x - \omega^h), m) \in \mathcal{X}^h(p)\}.$$

The total interest cost at end of period s is $\rho_s m_{ds}^h + r_s \bar{m}_s^h$; valued at beginning of period, it amounts to $(\rho_s m_{ds}^h + r_s \bar{m}_s^h)/(1 + \rho_s)$; that amount must be added to $p_s z_s^h$ in order to insert the total cost of the transactions z_s in the budget constraint for s .⁴

The prices $p_{\ell 0}$ of commodities $\ell \in L^I$ in period 0 are subject to lower bounds $\tilde{p}_{\ell 0} > 0$, entailing the possibility of excess supply – to be eliminated

³By term deposits in period 1.

⁴If transactions $pz > 0$ occur at a constant rate within the period, then interest charges $\rho m_d + r \bar{m}$ are minimal with $m_d = pz(r - \rho)/r$ and $\bar{m} = pz \rho/r$.

by quantity constraints on net sales. For household h , the net supply of commodity $\ell 0$ is $-z_{\ell 0}^h = \omega_{\ell 0}^h - x_{\ell 0}^h$. A constraint (upper bound) on net supply takes the form $-z_{\ell 0}^h \leq -\ell_{\ell 0}^h$, i.e. $x_{\ell 0}^h \geq \ell_{\ell 0}^h + \omega_{\ell 0}^h$, $-\omega_{\ell 0}^h \leq \ell_{\ell 0}^h \leq 0$. Similarly, a constraint on the net supply of commodity $\ell 0$ by firm j will take the form $y_{\ell 0}^j \leq \ell_{\ell 0}^j$. The maximal market net supply of commodity $\ell 0$ is thus $\sum_j \ell_{\ell 0}^j - \sum_h \ell_{\ell 0}^h := \sigma_{\ell 0}$.

I assume here that excess supply of commodity $\ell 0 \in L^I$ is eliminated by individual quantity constraints $(\ell_{\ell 0}^h, \ell_{\ell 0}^j)$, all $h \in \mathcal{H}$ and $j \in \mathcal{J}$, obtained as primitive functions of, and adding up to $\sigma_{\ell 0}$.⁵ And I define

$$\hat{X}^h(p, \sigma) = \{(x, m) \in \tilde{X}^h(p) \mid x_0 \geq \ell_0^h + \omega_0^h\} := \hat{X}^h(\xi_0).$$

Incomplete preferences over X^h are modeled by the *preference correspondence* $P^h : X^h \rightarrow X^h$ defined by $P^h(x) = \{x' \in X^h \mid x' \succ x\}$, the subset of X^h collecting the elements x' identified as strictly preferred to x . A simple monotonicity assumption will entail that $P^h(x)$ is non-empty:

$$(A.1) \text{ If } x' \geq x \text{ with } x'_{10} > x_{10}, \text{ then } x' \in P^h(x).$$

Two conditions on preference correspondences enter the assumptions of the Shafer-Sonnenschein theorem. They will be required for preferences over elements of the household's feasible set, namely the intersection of $\hat{X}^h$ with the household's budget set. But it aids intuition to define and discuss these conditions first for X^h . The conditions boil down, in this introductory discussion, to:

⁵Simple examples are provided by uniform rationing, proportional rationing, predrawn random rationing, a.s.o..

(Ca) For each $x \in X^h$, $x \notin HP^h(x)$, where $HP^h(x)$ denotes the convex hull of P^h .

(Cb) P^h has an open graph in $X^h \times X^h$.

Under complete preferences, the standard assumption of convex preferences⁶ entails that the preference correspondence P^h is convex-valued, so that (Ca) holds.⁷ The standard continuity assumption on preferences requires that the sets $PI^h(x) := \{x' \in X^h \mid x' \succeq x\}$ be closed. This implies that $P^h(x)$ is open – cf. Debreu (1959, p. 57). Thus, (Ca) and (Cb) on X^h are weakenings of the standard convexity and continuity assumptions on household preferences; the main dimension of “weakening” lies with the allowance for *incomplete* preferences.

The role of the standard assumptions is to derive the properties of demand correspondences required to invoke Kakutani’s fixed-point theorem, namely convex-valuedness and upper hemi-continuity. Similarly, I want to invoke (Ca) and (Cb) for preferences over household decisions in period 0, i.e. choices of initial consumption *and portfolios*. Actually, the complexity of portfolio choices is my main motivation for modeling preference as incomplete. So I turn next to feasible sets for these decisions.

In the sequel, I shall work with the intersection of $\hat{X}(p, \sigma)$ with a hypercube (of length) K centered at 0 and containing all feasible allocations.

I assume, for all $h \in \mathcal{H}$:

(A.2.1) $\omega^h \in \text{int } X^h$, and $\omega_s^h \in \text{int } X_s^h(x_0) \forall x_0^h \in X_0^h, s = 1, \dots, S$.⁸

⁶ $x' \in P^h(x)$ implies $\lambda x' + (1 - \lambda)x \in P^h(x)$ for all $\lambda \in (0, 1)$.

⁷Indeed, every element of $HP^h(x)$ is strictly preferred to x , hence different from x .

⁸This assumption is definitely unrealistic. It is widely used, because we know how it

(A.2.2) The correspondences $\chi^h(p)$, $\tilde{X}^h(p)$ and $X_s^h(x_0)$ are convex-valued and continuous; their intersections with K are closed-valued.

(A.2.3) The correspondence $\chi^h(p)$ is linear homogeneous in pz and m ; that is, if $(z, m) \in \chi^h(p)$, then $(z, \lambda m) \in \chi^h(\lambda p)$ and $(\lambda z, \lambda m) \in \chi^h(p)$.

2.2. Households receive signals $\xi_0 = (\xi_0^1, \xi_0^2) = ((p_0, r_0, \pi, \varphi, \sigma_0), ((V_0^j)_{j \in \mathcal{J}}, V_0^0), (y_0^j)_{j \in \mathcal{J}})$, where $\varphi = \frac{1}{1+\rho_0}$; these signals define the initial budget constraints

$$\begin{aligned} B_0^h(\xi_0) = & p_0 x_0 + \frac{\rho_0 m_{d0} + r_0 \bar{m}_0}{1 + \rho_0} + \sum_j \theta^{hj} (\pi_j - V_0^j) + \varphi b \\ & - [p_0 \omega_0^h + \pi \bar{\theta}^h + \bar{\theta}^{h0} V_0^0] \leq 0. \end{aligned} \quad (2.1)$$

The decision $d_0^h := (x_0, m_0, \theta, b)$ in $D_0^h := \tilde{X}_0^h \times [0, 1]^J \times R$ must in addition satisfy $(x_0, m_0) \in \hat{X}^h(p_0)$. I write henceforth $\hat{B}_0^h(\xi_0)$ for the constrained budget set incorporating the supply constraints.

The feasible set for d_0^h is thus

$$F_0^h(\xi_0) := \{d_0^h \in D_0^h \cap K \mid \hat{B}_0^h(\xi_0) \leq 0\}. \quad (2.2)$$

Vector inequalities: $a \geq b$ iff $a_i \geq b_i \forall i$; $a > b$ iff $a \geq b$ and $a \neq b$; $a \gg b$ iff $a_i > b_i \forall i$.

Lemma 1 *The correspondence defined by (2.2) is continuous for $p_0^I > 0$.*

Proof. (2.2) defines the feasible set $F_0^h(\xi_0)$ as the intersection of three correspondences, namely the budget set (2.1), the consumption-and-transactions set $\tilde{X}_0^h(\xi_0) \cap K$, and the set, say $\Gamma_0^h(\sigma_0)$, defined by the quantity constraints

could be generalized – at some inconvenience; see Arrow and Hahn (1971).

$x_0 \geq \ell_0^h(\sigma_0) + \omega_0^h$. It is convenient to combine first the inequality in (2.1) with Γ_0^h into the constrained budget set $\hat{B}_0^h(\xi_0)$.

Then, $F_0^h(\xi_0) = \hat{B}_0^h(\xi_0) \cap \tilde{X}_0^h(\xi_0)$. $\hat{B}_0^h(\xi_0)$ is continuous in ξ_0 when $p_0^I > 0$;⁹ and $\tilde{X}_0^h$ is continuous by assumption (A.2.2); they are both closed-valued; so their intersection is u.h.c. (Hildenbrand, 1974, Proposition 2.(a), p. 23). With $\omega_0^h \in \text{int } X_0^h$, as per assumption (A.2.1), the interiors of both $\hat{B}_0^h(\xi_0)$ and $\tilde{X}_0^h(\xi_0)$ include all $x \in X_0^h$ with $x \leq \omega_0^h$ and a continuum of m -values. The intersection of these two convex interiors is non-empty, and the intersection of the two correspondences is l.h.c. (Hildenbrand, 1974, Problem 6-(2), p. 25), hence continuous. ■

Households know that they will receive tomorrow in state s the signal $\xi_s^1 = (p_s, r_s, \varphi_s, \sigma_s)$. Whereas (p_s, σ_s) reflect market clearing, r_s and $\varphi_s = \frac{1}{1+\rho_s}$ will be set by the bank – possibly according to a pricing rule (an interest rule) whereby r_s, ρ_s are continuous functions of ξ_0 .

In order to define the prospective budget constraints of the households tomorrow in state s , the signals ξ_s^1 must be extended to profits of the firms and of the bank, $((V_s^j)_{j \in J}, V_s^0)$. Let $\xi_s := (\xi_s^1, V_s^1, \dots, V_s^J, V_s^0) := (\xi_s^1, \xi_s^2)$.

Using the profit levels V_s^j , one defines the budget constraints of the households tomorrow as

$$\begin{aligned} B_s^h(\xi_s \mid \xi_0, d_0^h) &= p_s x_s + \frac{\rho_s m_{ds} + r_s \bar{m}_s}{1 + \rho_s} \\ &- [p_s \omega_s^h + \sum_j \theta^{jh} V_s^j + b^h + \bar{\theta}^{h0} V_s^0] \leq 0. \end{aligned} \quad (2.3)$$

The feasible set for household h at s is well-defined given d_0^h and $\xi_s =$

⁹See the lemma in section 3 of Drèze (1975).

(ξ_s^1, ξ_s^2) as

$$F_s^h(\xi_s \mid \xi_0, d_0^h) := \{(x_s, m_s) \in \hat{X}_s^h(x_0, p_s, \sigma_s) \cap K \mid \hat{B}_s^h(\xi_s \mid \xi_0, d_0^h) \leq 0\}, \quad (2.4)$$

where

$$\hat{B}_s^h(\xi_s \mid \xi_0, d_0^h) := \{(x_s, m_s) \mid B_s^h(\xi_s \mid \xi_0, d_0^h) \leq 0, x_s \leq \ell_s^h(\sigma_s) + \omega_s^h\} \cap K. \quad (2.5)$$

By a straightforward transposition of the proof of Lemma 1, one verifies:

Lemma 2 *The correspondence defined by (2.4) is continuous for $p_s^I > 0$.*

$F_s^h(\xi_s \mid \xi_0, d_0^h) \cap K$ is a convex, compact set. Upon observing ξ_s , household h will select $(x_s, m_s) \in F_s^h(\xi_s \mid \xi_0, d_0^h)$ such that $P^h(x_s \mid x_0^h) \cap F_s^h(\xi_s \mid \xi_0, d_0^h) = \emptyset$. I denote such a selection by $x_s^h(\xi_s \mid \xi_0, d_0^h)$; it needs not be unique.¹⁰ For given ξ_0 , the decision d_0^h thus entails a *consumption profile* $(x_0^h, x_s^h(\xi_s \mid \xi_0, d_0^h))$ defined for all states s and signals ξ_s . The preferences among decisions $d_0^h = (x_0^h, m_0^h, \theta^h, b^h) \in F_0^h(\xi_0)$ should thus reflect preferences among the associated consumption profiles.

I henceforth write $x(d_0^h)$ for a consumption profile associated with d_0^h .

2.3. The incomplete preferences of household h over current decisions $d_0^h = (x_0^h, m_0^h, \theta^h, b^h)$ will be represented by the preference correspondence $P^h(d_0^h \mid \xi_0) : (X_0^h, R_+^2, \Delta^J, R) \rightarrow (X_0^h, R_+^2, \Delta^J, R)$ defined by $P_0^h(d_0^h \mid \xi_0) = \{d_0 \in (X_0^h, R_+^2, \Delta^J, R) \mid d_0 \succ d_0^h \text{ given } \xi_0\}$, that will be assumed to satisfy

¹⁰Uniqueness is not required for the sequel. Under complete preferences over $x_s \in \hat{X}_s(x_0^h)$, alternative selections are mutually indifferent.

extensions of the conditions (Ca) and (Cb), justified again by reference to standard convexity and continuity properties of consumption preferences.

The standard approach in TGE¹¹ is to: (i) associate with a portfolio (θ, b) the collection of feasible sets $F_s^h(\xi_s \mid \xi_0, d_0^h)$; (ii) select for each ξ_s a consumption vector in $F_s^h(\xi_s \mid \xi_0, d_0^h)$ that is optimal given x_0^h ; (iii) assume that preferences among vectors $(x_0^h, x_s(\xi_s))$ are represented by an expected utility $v^h(d_0^h) := E_{\xi_s} u_s(x_0^h, x_s(\xi_s))$, where the expectation is computed with subjective probabilities $\psi^h(\xi_s \mid \xi_0)$ assumed unique, tight¹² and continuous.

It is worth noting that the separation of utilities and probabilities is arbitrary under state-dependent preferences.¹³ It is thus equivalent to express $v^h(d_0^h \mid \xi_0)$ as a function of vectors $(x_0^h, x_s(\xi_s))$. Convexity (and continuity) of preferences among such vectors is part of the standard approach. Note also that convexity of preferences among pairs of vectors like $(x_0^h, x_s(\xi_s), x_s(\xi'_s))$ and $(x_0^h, x'_s(\xi_s), x'_s(\xi'_s))$ amounts to risk aversion.

The link between complete consumption preferences and preferences over decisions $d_0^h = (x_0^h, m_0^h, \theta^h, b^h) \in F_0^h(\xi_0)$ is fairly transparent when the signals ξ_s belong to finite sets Ξ_s ¹⁴ – a feasible specification given that TGE does not entail market clearing prices tomorrow. Let indeed d and d' belong to $F_0^h(\xi_0)$ with $d \succ d'$, and consider the consumption profiles $x(d), x(d')$ and $\lambda x(d) + (1 - \lambda)x(d')$, $\lambda \in (0, 1)$. Under complete con-

¹¹See Grandmont (1977, 1988), Jordan (1977) or Sondermann (1974).

¹²“Tight”: for every $\varepsilon > 0$, there exists a compact subset $C \subset R^{S(2L+1)}$ such that $\psi^h C \mid \xi_0 \geq 1 - \varepsilon$ identically in ξ_0 .

¹³See Drèze and Rustichini (2004) for a survey of this issue.

¹⁴This is assumed, for instance, by Balasko (2003, p. 2) “for the sake of simplicity ... though it would be more appropriate, if technically more demanding, to assume this set to be infinite”. Under incomplete preferences, a finite support is natural.

vex consumption preferences, $\lambda x(d) + (1 - \lambda)x(d') \succ d'$. Consider next the decision $d^\lambda = \lambda d + (1 - \lambda)d' = (\lambda x_0 + (1 - \lambda)x'_0, \lambda m_0 + (1 - \lambda)m'_0, \lambda \theta + (1 - \lambda)\theta', \lambda b + (1 - \lambda)b')$. By (A.2.3), $(x_0^\lambda, m_0^\lambda) \in \tilde{X}^h(\xi_0)$, so that $d^\lambda \in F_0^h(\xi_0)$; and for all (s, ξ_s) , $\lambda x_s(\xi_s \mid \xi_0, d) + (1 - \lambda)x_s(\xi_s \mid \xi_0, d') \in F_s^h(\xi_s \mid \xi_0, d^\lambda)$, because $B_s^h(\xi_s \mid \xi_0, \cdot)$ is linear in (θ, b) . Accordingly, for each (s, ξ_s) , $(x_0^\lambda, x_s(\xi_s \mid \xi_0, d_0^\lambda)) \succeq \lambda x_s(\xi_s \mid \xi_0, d) + (1 - \lambda)x_s(\xi_s \mid \xi_0, d')$, so that $d^\lambda \succeq \lambda x(d) + (1 - \lambda)x(d') \succ d'$: preferences over the decisions $(d_0^h \mid \xi_0)$ are convex – and thus satisfy the condition (Ca).

This reasoning underlies my intuitive justification for imposing (Ca) directly on preferences over initial decisions d_0^h :

(CF_{0a}) Identically in ξ_0 and $d_0^h \in F_0^h(\xi_0)$, $d_0^h \notin HP_0^h(d_0^h \mid \xi_0)$.

The other property that I wish to assume for $P_0^h(d_0^h \mid \xi_0)$ is

(CF_{0b}) P_0^h has an open graph in $\Xi_0 \times F_0^h(\xi_0) \times D_0^h$ for $p_0^I > 0$.

This property can be spelled out as follows. For some ξ_0 , let d_0^h in $F_0^h(\xi_0)$ and $d_0 \in P_0^h(d_0^h \mid \xi_0)$. There exists an open neighborhood $\mathcal{B}(\xi_0, d_0^h, d_0) \in \Xi_0 \times F_0^h(\xi_0) \times D_0^h$ such that $(\xi, d^h, d) \in \mathcal{B}$ implies $d \in P_0^h(d^h \mid \xi)$. In other words, small variations in period 0 signals and/or decisions do not entail major variations in (incomplete) preferences. This of course requires that the sets $P_0^h(d_0^h \mid \xi_0)$ be open. Beyond that natural implication of standard continuity assumptions, it is also required here that the feasible set $F_0^h(\xi_0)$ be continuous, as verified in Lemma 1 for $p_0^I > 0$ – hence the caveat in (CF_{0b}).

2.4. The analysis of initial decisions often concentrates on the portfolio variables (θ^h, b^h) under fixed values for (x_0^h, m_0^h, ξ_0) . The derivation of portfolio

preferences from consumption preferences rests upon a standard concept: the *indirect utility function* defined on incomes and prices.

Under given consumption x_0^h , the preferences over portfolios (θ^h, b^h) may be interpreted as reflecting the indirect utilities of income at node 0 given ξ_0 (hence p_0) and at nodes s given ξ_s (hence p_s). Indeed, a given portfolio (θ^h, b^h) has a cost today $(\pi\theta^h + \varphi b^h)$ and yields incomes tomorrow $(\theta^h V + b^h)$. Accordingly, preferences over income streams are implicit in preferences over portfolios (over current decisions) – hence satisfy (CF₀a) and (CF₀b).

In Section 3, I shall be concerned with preferences over (marginal) variations in incomes under given initial signals ξ_0 and decisions $d_0^h(\xi_0)$. The relevant definitions of “incomes” are then:

$$\begin{aligned} W_0^h(\xi_0 \mid d_0^h) &= p_0 \omega_0^h + \pi \bar{\theta}^h + (V_0 - \pi) \theta^h - \varphi b^h, \\ W_s^h(\xi_s \mid d_0^h) &= p_s \omega_s^h + b^h + V_s \theta^h + V_s^0 \bar{\theta}^{h0}. \end{aligned} \tag{2.6}$$

Variations in incomes will be denoted $\Delta W^h = (\Delta W_0^h, (\Delta W_s^h(\xi_s))_{s \in S, \xi_s \in \Xi_s})$. Preferences over such variations follow from preferences over the associated variations in consumption at the given prices $(p_0(\xi_0), p_s(\xi_s)_{s \in S})$. Standard assumptions would imply that preferences over ΔW^h are convex and continuous. I shall assume that they satisfy (Ca) and (Cb).

Let $P_W^h(W^h \mid \xi_0, d_0^h) : R^{1+\sum_s \Xi_s} \rightarrow R^{1+\sum_s \Xi_s}$ be defined by

$$P_W^h(W^h \mid \xi_0, d_0^h) := \{W \in R^{1+\sum_s \Xi_s} \mid W \succ W^h \text{ given } \xi_0, d_0^h\}.$$

(CWA) Identically in ξ_0 and $d_0^h \in F_0^h(\xi_0)$, $W^h \notin HP_W^h(W^h \mid \xi_0, d_0^h)$.

(CWB) P_W^h has an open graph in $\Xi_0 \times F_0^h(\xi_0) \times R^{1+\sum_s \Xi_s}$.

3 Firms

3.1. A firm j is defined by its production set $Y^j \subset R^{L(S+1)}$, by a set of $S+1$ transactions correspondences $\Psi_s^j(p_s)$, $s = 0, 1, \dots, S$ defining the feasible pairs (y_s^j, n_s^j) in function of the prices p_s , and by a correspondence $\mathcal{J}^j(\Theta) \subset \{1, 2, \dots, H\}$ defining the firm's "control group" (veto players for production decisions, more on this below). For each s , $n_s^j = (n_{ds}^j, \bar{n}_s^j)$. I define $\tilde{Y}^j(p) = \{(y, n) \mid y \in Y^j \cap K, (y, n) \in \Psi^j(p)\}$, $\hat{Y}^j(\xi_0) = \{(y, n) \in \tilde{Y}^j(p) \mid y_0^j \leq \ell_0^j\}$ and I assume

$$(B.1) \quad \forall j \in \mathcal{J}, Y^j \text{ is convex and closed; it satisfies free disposal: } -R_+^{L\bar{S}} \subset Y^j.$$

$$(B.2) \quad \forall j \in \mathcal{J}, \Psi^j(p) \text{ and } \tilde{Y}^j(p) \text{ are continuous.}$$

For every $s \in S$ and $y_0^j \in Y_0^j$, these properties are inherited by $Y_s^j(y_0^j)$ and $\tilde{Y}_s^j(y_0^j, p_s)$. Also, $\hat{Y}^j(\xi_0)$ is continuous.¹⁵

In the GEI approach, a production decision for firm j is an $(L+2)(S+1)$ vector $(y^j, n^j) = ((y_0^j, n_0^j), \dots, (y_s^j, n_s^j), \dots)$ defining uniquely the production (and liquidity) plan for today (y_0^j, n_0^j) and for tomorrow under every state $(y_s^j, n_s^j)_{s=1, \dots, S}$. Because all agents share common correct point expectations about equilibrium prices in every state $s = 1, \dots, S$, the profit maximising period 1 plans can be decided unequivocally at node 0.

In the TGE approach developed here, where price expectations for tomorrow under state s are idiosyncratic and probabilistic, the decision at node 0 concerns the initial vector (y_0^j, n_0^j) and the conditional production

¹⁵See Dehez and Drèze (1984, footnote 8).

sets $Y_s^j(y_0^j)$ defined by the projections of Y^j on the subspace containing y_s^j , given y_0^j . That is, the firm chooses current outputs and inputs (including investments) and the S production sets constraining its activity under state s , $Y_s^j(y_0^j)$, $s = 1, \dots, S$. After observing ξ_s^1 , the firm will choose $(y_s^j, n_s^j) \in \tilde{Y}_s^j(y_0^j, p_s)$ so as to maximise profits $p_s y_s^j + \frac{\rho_s n_{ds}^j + r_s \bar{n}_s^j}{1 + \rho_s} = V_s^j(\xi_s^1)$. Since $\tilde{Y}_s^j(y_0^j)$ is convex and closed, $V_s^j(\xi_s^1)$ is a continuous function of ξ_s^1 . If the range of the relevant elements of p_s is wide, the firm will privilege “flexibility” under $Y_s^j(y_0^j)$ – a consideration absent under GEI.¹⁶

I am not aware of any satisfactory published discussion of decision criteria for business firms under TGE. The approach used here rests upon Drèze (1989), where a reasonably realistic decision criterion for firms is introduced, that lends itself to a TGE extension. That criterion, labeled “control principle”, works as follows (in the present context):

- (i) for each firm j , with every vector of shareholdings $(\theta^{hj})_{h \in H}$ is associated a subset of shareholders $\mathcal{I}^j$, called “control group”;
- (ii) a feasible plan $((y_0^j, n_0^j), (Y_s^j(y_0^j))_{s=1, \dots, S}) := d^j$ is preferred by firm j to the alternative feasible plan $\hat{d}^j$ if and only if it is preferred by a set of shareholders $\hat{\mathcal{I}}^j \subset \{1, 2, \dots, H\}$ with $\sum_{h \in \hat{\mathcal{I}}^j} \theta^{jh} > \frac{1}{2}$ and $\mathcal{I}^j \subseteq \hat{\mathcal{I}}^j$.

As an illustration, think of $\mathcal{I}^j$ as the “board of directors” for firm j . A modification of the firm’s production plan requires approval by every board member *and* by a majority of shareholders. So, the director(s) is (are) veto player(s), and majority voting is immune to the Condorcet paradox.¹⁷

¹⁶I allude here to “flexibility given state s ”. Under GEI, there remains scope for “flexibility across states” – a different notion (also present here).

¹⁷Thus, the “control group” is not itself a primitive, as it is allowed to reflect equilibrium

The resulting decision criterion defines only a partial ordering of production plans, leaving room for multiple equilibria. But that partial ordering satisfies the two conditions required for reliance upon the Shafer-Sonnenschein (1975) existence theorem.¹⁸

Remark 3.1. Under incomplete preferences, it would be natural to replace the simple majority rule $\sum_{h \in \mathcal{I}^j} \theta^{hj} > \frac{1}{2}$ by the weaker requirement $\sum_h (\theta^{hj} \mid d^j \in P^{hj}(\hat{d}^j)) > \sum_h (\theta^{hj} \mid \hat{d}^j \in P^{hj}(d^j))$. But the resulting preference correspondences for firms would not have an open graph. This point deserves further attention.

Existence of an equilibrium for the control principle rests on a simple assumption about the control group.

(CC) **Continuity of control:** For all h and j , the set of ownership matrices Θ such that $h \in \mathcal{I}^j(\Theta)$ is closed.¹⁹

3.2. In order to use the “control principle”, one must specify the meaning of the statement: “ d^j is preferred to $\hat{d}^j$ by shareholder h ”.

shareholdings (endogenous). Only the statutory rule defining the control group on the basis of equilibrium shareholdings is exogenous. As noted in Drèze (1989), the approach can readily be extended to multistage control (e.g. majority voting inside the control group) and to (partial) delegation of authority to the board, or to management; also how absentee ownership could be reflected in the majority rule; a.s.o..

¹⁸See Appendix 2 in Drèze (1989).

¹⁹Examples: all shareholders owning at least $\alpha\%$ of the shares; the n leading shareholders; the leading shareholders owning together $\beta\%$ of the shares; a.s.o.. Alternative statement of (CC): For all j , the correspondence $\mathcal{I}^j(\Theta)$ s upper hemi-continuous for the discrete topology. See Drèze (1989, Appendix 2).

I recall that a decision d^j of firm j entails a family of dividends $(V_0^j((y_0^j, n_0^j) \mid \xi_0^1), (V_s^j(y_0^j, \xi_s^1), s = 1, \dots, S))$ of which agent h collects a fraction θ^{jh} . The implications for h of a change from d^j to $\hat{d}^j$ are captured by the family of associated changes in dividends, of which a fraction θ^{jh} appears in the incomes $W_s^h(\cdot)$.

Thus, replacing the decision δ^j by the feasible alternative $\hat{\delta}^j$ entails for shareholder h the collection of “income effects”

$$\begin{aligned} \Delta W_0^h &= W_0^h(\hat{\delta}^j) - W_0^h(\delta^j) = \theta^{hj}(\hat{V}_0^j - V_0^j) \\ \Delta W_s^h(\xi_s^1) &= W_s^h(\xi_s^1 \mid \hat{\delta}^j) - W_s^h(\xi_s^1 \mid \delta^j) = \theta^{hj}[\hat{V}_s^j(\xi_s^1) - V_s^j(\xi_s^1)], \end{aligned} \quad (3.1)$$

where $\hat{V}_0^j - V_0^j = p_0(\hat{y}_0^j - y_0^j) + \frac{\rho_0(\hat{n}_{d0}^j - n_{d0}^j) + r_0(\hat{n}_0^j - \bar{n}_0^j)}{1 + \rho_0}$ and likewise for $\hat{V}_s^j - V_s^j$, $s = 1, \dots, S$.

Accordingly, initial shareholder h will favor the replacement of δ^j by $\hat{\delta}^j$ and only if the income effects $(\Delta W_0^h, \Delta W_s^h(\xi_s^1))$ defined in (3.1) are desirable (are preferred to the status quo); that is:

$$W^h + \Delta W^h \in P_W^h(W^h \mid \xi_0, d_0^h).$$

Hence:

$$\hat{d}^j \in P_h^j(d^j) \text{ iff } W^h(\hat{d}^j) \in P_W^h(W^h(d^j) \mid \xi_0, d_0^h). \quad (3.2)$$

Condition (3.2) may be inserted into condition (ii) defining the “control principle”. Before commenting further on the rationale of (3.2), I note that if $P_h^j(d^j)$ satisfies properties (Ca) and (Cb) above for every shareholder h of firm j (every h with $\theta^{jh} > 0$) – see Lemma 3 below – then the partial ordering of d^j ’s resulting from the control principle will also verify (Ca) and (Cb) – see Lemma 4 below.

It thus follows from assumptions (CWa) and (CWb) that:

Lemma 3 $P_h^j(d^j)$ as defined by (3.2) satisfies properties (Ca) and (Cb).

Lemma 4 $P^j(d^j)$ as defined by the control principle and (3.1) satisfies properties (Ca) and (Cb).

Proof.

Ad (Ca). Let $h \in \mathcal{I}^j$. Then, $P^j(d^j) \subseteq P_h^j(d^j)$. Hence $H P^j(d^j) \subseteq H P_h^j(d^j)$. Because $d^j \notin H P_h^j(d^j)$, it follows that $d^j \notin H P^j(d^j)$.

Ad (Cb). $P^j(\cdot)$ has open graph if and only if, for all d^j and $\hat{d}^j \in P^j(d^j)$, there exists a neighborhood $B(d^j, \hat{d}^j)$ such that all elements $(d', \hat{d}')$ of $B(d^j, \hat{d}^j)$ satisfy $\hat{d}' \in P^j(d')$.

According to the control principle, $\hat{d}^j \in P^j(d^j)$ implies the existence of a set of shareholders $\hat{\mathcal{I}}^j, \mathcal{I}^j \subseteq \hat{\mathcal{I}}^j$, $\sum_{h \in \hat{\mathcal{I}}^j} \theta^{jh} > \frac{1}{2}$, such that $\hat{d}^j \in P_h^j(d^j) \forall h \in \hat{\mathcal{I}}^j$. Because $P_h^j(\cdot)$ has open graph (Lemma 3), $\hat{d}' \in P_h^j(d')$ for all h in $\hat{\mathcal{I}}^j$; this entails $\hat{d}' \in P^j(d')$. It is shown in Drèze (1989, p. 127) that $P^j(\cdot)$ is robust to rescalings of θ^j when $\pi^j = 0$ with $\sum_h \theta^{hj} < 1$. ■

4 Existence theorem

Definition 4.1 A TGE for the economy described above is defined by:

- a set of decision rules $(\delta^h)_{h \in \mathcal{H}}$ and $(\delta^j)_{j \in J}$ with node 0 components d_0^h, d_0^j ;
- a set of node 0 signals $\xi_0^1 = (p_0, r_0, \pi, \varphi, \sigma_0, V_0^0)$,

such that

$$(i) \forall h, d_0^h \in F_0^h(\xi_0) \text{ and } F_0^h(\xi_0) \cap P^h(\delta^h) = \emptyset;$$

(ii) $\forall j, d_0^j \in \hat{Y}_0^j(\xi_0)$ and $\hat{Y}_0^j(\xi_0) \cap P^j(\delta^j) = \emptyset$;

(iii) $\sum_h b^h = 0$;

(iv) $\sum_h \theta^{jh} = 1, j = 1, \dots, J$;

(v) $V_0^0 = \sum_h \frac{\rho_0 m_{d0}^h + r \bar{m}_0^h}{1 + \rho_0} + \sum_j (\sum_h \theta^{jh}) \frac{\rho_0 n_{d0}^j + r \bar{n}_0^j}{1 + \rho_0}$;

(vi) $\sum_h (x_0^h - \omega_0^h) - \sum_j (\sum_h \theta^{jh}) y^j \leq 0$;

(vii) $p_{\ell 0} > \tilde{p}_{\ell 0}, \ell \in \mathcal{L}^{II}$, implies

$$x_{\ell 0}^h > \ell^h(\sigma_{\ell 0}) + \omega_{\ell 0}^h \text{ and } y_{\ell 0}^j < \ell^j(\sigma_{\ell 0}).$$

Theorem 4.2 *Under the assumptions (A.1), (A.2), (CF₀a), (CF₀b), (CWA), (CWB) (B.1), (B.2) and (CC), for every $\bar{A} \in R_{++}$, there exists a TGE with $\sum_{\ell} p_{\ell 0} \tilde{\sigma}_{\ell 0} = \bar{A}, \tilde{\sigma}_{\ell 0} = \frac{\sigma_{\ell 0}}{K}$.*

Define $\bar{u} := \sum_{\ell} (1 - \frac{\sigma_{\ell 0}}{K}) p_{\ell 0} / \sum_{\ell} p_{\ell 0}$ = weighted average of relative aggregate supply limitations $\frac{K - \sigma_{\ell 0}}{K} \in [0, 1)$ – one, among several, measure of “output gap”. Since $\bar{A}$ in Theorem 4.2 is arbitrary (positive), we may divide that number by the sum of past prices $\sum_{\ell} p_{\ell(-1)}$ and write A for the quotient. Define $1 + i_0 := \sum_{\ell} p_{\ell 0} / \sum_{\ell} p_{\ell(-1)}$, so that i_0 is the rate of inflation for period 0. The conclusion of Theorem 4.2 may then be rewritten as:

$$\text{For every } A \in R_{++}, \text{ there exists a TGE with } (1 + i_0)(1 - \bar{u}) = A. \quad (4.1)$$

The interpretation of (4.4) is discussed at some length in Drèze (2013, section 5.3). I limit myself here to quote “an intriguing implication: formula (4.4)²⁰ allows for two distinct tradeoffs between inflation and unemployment!

Indeed:

²⁰Formula (5.5) in Drèze (2013).

- keeping A fixed, $\bar{u}$ and i_0 can move in parallel, either increasing (stagflation) or decreasing (recovery) together;
- under variations of A , $\bar{u}$ and i can move in opposite directions (Phillips curve)."

Proof of Theorem 4.2.

Following standard practice, I compactify the domains of the decision rules δ^h , δ^j to belong to a hypercube centered at 0 and with length K , a cube containing all feasible allocations.

In Part I of the proof, I impose $\theta^{jh} \geq \varepsilon \bar{\theta}^{jh}$, $\varepsilon > 0$, and let $\varepsilon \rightarrow 0$ in Part IIa. Also, I impose $\pi \in [0, G]^J$ for some large, arbitrary scalar G in R_{++} , and let $G \rightarrow \infty$ in Part IIb. To handle $\sum_h \theta^{jh} \neq 1$, I proceed as follows:

- (i) if $\pi_j = 0$ with $\sum_h \theta^{jh} < 1$, I rescale (y_0^j, n_0^j) by $\sum_h \theta^{jh}$ and θ^{jh} by $1/\sum_h \theta^{jh}$; this leaves dividends per shareholder unchanged, but enters the formulation of conditions (v) and (vi) .
- (ii) if $\pi_j = G$ with $\sum_h \theta^{jh} > 1$, I introduce an aggregate demand constraint on shareholdings γ_j , translated into individual constraints $\ell^j(\gamma_j) > 0$, $\theta^{jh} \leq \ell^j(\gamma_j)$, where the functions ℓ^j are single-valued continuous.

Part I.

I.a. I consider the abstract economy defined by $H + J + 1$ agents:

- $h = 1, \dots, H$, with decisions $\delta^h(\xi_0)$ of period 0 components $d_0^h \in F_0^h(\xi_0)$ and preferences defined by $P_h(\delta^h)$;
- $j = H + 1, \dots, H + J$, with decisions $\delta^j(\xi_0^1)$ of period 0 components $(y_0^j, n_0^j) \in \hat{Y}_0^j(\xi_0)$ and preferences defined by $P_j(\delta^j)$;

- $n = H + J + 1$, the market agent, with decision (q, ψ, φ) and preferences

$P_n(q, \psi, \varphi)$ defined by

$$P_n(q, \psi, \varphi) = \left\{ (q', \psi', \varphi') \in \overline{A}\Delta^L \times [0, 2G]^J \times [0, 1] \right. \\ \left. \mid \sum_{\ell} (q'_{\ell} - q_{\ell}) \left[\sum_h (x_0^h - \omega_0^h) - \sum_j (\sum_h \theta^{hj}) y_0^j \right] \right. \\ \left. + \sum_j (\psi'_j - \psi_j) (\sum_h \theta^{jh} - 1) + (\varphi' - \varphi) \sum_h b^h > 0 \right\}.$$

It is immediate that $P_n(\cdot)$ has open graph, and that $(q, \psi, \varphi) \notin HP(q, \psi, \varphi)$, a convex set. Also, the domain of $P_n(\cdot)$ is (constant, hence) continuous. As for $P_h(\cdot)$ and $P_j(\cdot)$, similar properties have been established in Sections 2 and 3. As for the side-condition “ $p_0^I > 0$ ” in Lemma 1, it is shown below that it is always satisfied at equilibrium, so that the device in Greenberg (1977) permits ignoring that condition.

The bank is not introduced as a separate agent, but it is understood that V_0^0 is defined by (v) in Definition 4.1 and V_s^0 is part of $\xi_s \forall s \in S$.

The prices p_0 and the aggregate supply constraints σ_0 are defined as follows, where $\tilde{p}_{\ell_0} \geq 0$ denotes the lower bounds on p_{ℓ_0} (as part of the primitives):

$$\text{on } \mathcal{L}^I, \quad p_{\ell_0} = q_{\ell} \text{ and } \sigma_{\ell_0} = K; \quad (4.2)$$

$$\text{on } \mathcal{L}^{II}, \quad p_{\ell_0} = \max(q_{\ell}, \tilde{p}_{\ell_0}) \text{ and } \sigma_{\ell_0} = \frac{K}{\tilde{p}_{\ell_0}} \min(q_{\ell}, \tilde{p}_{\ell_0}). \quad (4.3)$$

Similarly the prices π_j and the aggregate demand constraints γ_j are defined as follows:

$$\pi_j = \min(\psi_j, G), \gamma_j = K \left\{ 1 - \frac{\max(\psi_j, G) - G}{G} \right\} \in [0, K]. \quad (4.4)$$

The aggregate demand constraint γ_j is transposed to households h through continuous functions.

Applying the Shafer-Sonnenschein theorem, the abstract economy has an equilibrium. I next show that it is a temporary equilibrium as per Definition 4.1 for the economy of this Part I, and label it a (G, ε) -equilibrium. It departs from an equilibrium for Definition 4.1 in two respects: the constraints $\theta^{hj} \geq \varepsilon \bar{\theta}^{hj}$ and the allowance for demand rationing on the asset markets (when $\gamma_j < K$).

I.b.

1. From the preferences of the market agent, it follows that $\varphi \sum_h b^h \geq 0$, because otherwise $\varphi = 0$ would be preferred by the market agent.
2. From the preferences of the market agent, it follows that $\sum_j \pi_j (\sum_h \theta^{jh} - 1) \geq 0$. Indeed, $\pi_j \geq 0 \forall j$ by construction, since $\pi_j \geq \min(\psi_j, G) \geq 0$. If $\sum_h \theta^{jh} - 1 < 0$, the market agent chooses $\psi_j = 0$, implying $\pi_j = 0$.
3. $\sum_h B_0^h = pz + \sum_j \pi_j (\sum_h \theta^{jh} - 1) + \varphi \sum_h b^h = \tau_1 + \tau_2 + \tau_3 \leq 0$. Since $\tau_2 \geq 0$ by (2) and $\tau_3 \geq 0$ by (1), it must be that $\tau_1 \leq 0$.
4. $p_1 = q_1 > 0$.

Indeed, if $p_1 = q_1 = 0$, then $z_1 = K$ by strict monotonicity (A.1), and for all ℓ , $q_\ell > 0$ implies $z_\ell = K$, so that $qz = \bar{A}K$.

By construction, $p_\ell \geq q_\ell \forall \ell \in \mathcal{L}$. If there exists $\ell \in \mathcal{L}^I$ with $z_\ell < 0$, then $q_\ell = p_\ell = 0$. If there exists $\ell \in \mathcal{L}^{II}$ with $z_\ell < K$, then $q_\ell = 0$, $p_\ell = \tilde{p}_\ell > 0$ and $\sigma_\ell = 0$ implying $z_\ell \geq 0$. Thus $p_\ell z_\ell \geq q_\ell z_\ell \forall \ell \in \mathcal{L}$ and $pz \geq qz = \bar{A}K$ – contradicting 3 above.

With $p_1 = q_1 > 0$, $\sum_h B_0^h = 0$.

5. $z_\ell \leq 0$ and $p_\ell z_\ell = 0 \forall \ell \in \mathcal{L}$.

Indeed, if there exists $\bar{\ell}$ with $z_{\bar{\ell}} > 0$, then $q_\ell = 0$ for all ℓ with $z_\ell < z_{\bar{\ell}}$ and $qz \geq z_{\bar{\ell}}\bar{A} > 0$. Repeating the reasoning under 4, $pz \geq z_{\bar{\ell}}\bar{A} > 0$, contradicting again 3 above. If $z_\ell < 0$, then $q_\ell = 0$. If $\ell \in \mathcal{L}^I$, $p_\ell = 0$; if $\ell \in \mathcal{L}^{II}$, $q_\ell = 0$ implies $\sigma_\ell = 0$, $z_\ell \geq 0$ hence $z_\ell = 0$. Thus, $p_\ell z_\ell = 0 \forall \ell \in \mathcal{L}$.

6. Returning to 1 and 3 with $\sum_h B_0^h = 0$ and $pz = 0$, it follows that $\varphi \sum_h b^h + \sum_j \pi_j (\sum_h \theta^{jh} - 1) = 0$, hence each of these two terms is equal to 0. This implies either $\sum_h \theta^{jh} = 1$ or $\sum_h \theta^{jh} - 1 < 0$ with $\pi_j = 0$. Indeed, $\sum_h \theta^{jh} - 1 > 0$ with $\pi_j > 0$, hence $\psi_j > 0$, would imply $\sum_j \pi_j (\sum_h \theta^{jh} - 1) > 0$, contradicting 3.

7. When $z_\ell < 0$ (with $p_\ell = 0$), z_ℓ may be set equal to 0 by free disposal.

When $\sum_h \theta^{jh} < 1$, one may rescale (y^j, n^j) by a positive scalar equal to $\sum_h \theta^{jh}$. The rescaled plan remains feasible because $0 \in Y^j$. Multiplying parallelly θ^{jh} by $1/\sum_h \theta^{jh}$, the dividends of every shareholder are unchanged. This last adjustment was already part of conditions (v) - (vi); it was implicit in the definitions underlying the proof, and explicit in the definition of the preferences of the market agent.

I have thus established that an equilibrium of the constrained economy $\mathcal{E}(G, \varepsilon)$ is a temporary equilibrium of that economy, in so far as conditions (i) - (vi) are concerned.

8. That conditions (vii) are also satisfied is immediate. Indeed, (4.3) implies $\sigma_\ell = K$ whenever $p_\ell = q_\ell < \tilde{p}_\ell$. A similar property holds for portfolios, since (4.4) implies $\gamma_j = K$ whenever $\pi_j = \psi_j < G$. Thus,

for every $G > 0$, there exists a (G, ε) -equilibrium.

Part II.

II.a. As a first step, I wish to eliminate the restriction $\theta^{jh} \geq \varepsilon \bar{\theta}^{jh}$, introduced above to guarantee $\sum_h \theta^{jh} > 0$ and permit the rescaling of portfolios by a finite scalar $1/\sum_j \theta^{jh}$.

Consider the sequence of (G, ε^ν) -equilibria indexed by $\nu = 1, 2, \dots$, $\lim_{\nu \rightarrow \infty} \varepsilon^\nu = 0$, and defined by the conditions $\theta^{jh} \geq \varepsilon^\nu \bar{\theta}^{jh}$. We wish to show that a limit of the (G, ε^ν) -equilibria exists, and is itself a G -equilibrium.

A (G, ε^ν) -equilibrium is a tuple $a^\nu = (p^\nu, \pi^\nu, \varphi^\nu, \frac{\sigma^\nu}{K}, \frac{\psi^\nu}{K}, z^\nu, \Theta, \sum_h b^{h\nu}, V^{o\nu})$ in $\bar{A}\Delta^L \times [0, G]^J \times [0, 1] \times [0, 1]^L \times [0, 1]^J \times [-K, +K]^L (\Delta^J)^J \times [-\bar{A}K - G, +\bar{A}K + G]H \times [0, \bar{A}K]_K$. Thus every element of a is bounded, and there exist convergent subsequences. I must show that every limit point of such a subsequence is a G -equilibrium for $\varepsilon = 0$.

It is readily verified that only conditions (i) and (ii) in Definition 4.1 require verification. The other conditions involve linear inequalities or equalities that are preserved at the limit. Now both $P^h(\cdot)$ and $P^j(\cdot)$ have open graph. So, if conditions (i) and (ii) hold for *every* $\varepsilon > 0$, they also hold for $\varepsilon = 0$.

II.b. Consider now an increasing sequence of values $G^1, \dots, G^k, \dots$ in R_+ with $\lim_{k \rightarrow \infty} G^k = +\infty$. For each finite G^k there exists a temporary equilibrium (with quantity constraints on the demand for shares), say $a^k = (p^k, \pi^k, \varphi^k, \frac{\sigma^k}{K}, \frac{\psi^k}{K}, z^k, V^{ok})$. All the components of a^k , other than ψ^k , belong to compact sets. Hence, there exist for these convergent subsequences. For any such subsequence, if π^k also converges, then the limit is a

temporary equilibrium according to Definition 4.1. Indeed, let the limit correspond to G^N , so that for each j we have $\pi_j^N \leq G^N$. That limit also defines a $(N + 1)$ -equilibrium with $\pi_j^{N+1} \leq G^N < G^{N+1}$, which implies $\gamma_j^{N+1} = K$, so that the $(N + 1)$ -equilibrium satisfies Definition 4.1.

Suppose instead that π^k diverges to infinity along the converging subsequence, so that the relative prices of commodities (adding up to $\bar{A}$) and of bonds ($\varphi \leq 1$) in terms of asset prices tend to 0. The budget sets of households remain continuous at the limit prices (0), because: (i) initial endowments of commodities are strictly positive; (ii) net trades in bonds have arbitrary signs, so that a 0 price does not entail a discontinuity.

Along the relevant subsequence, $z \leq 0$ ($= 0$ after free disposal) and $\sum_h b^h = 0$ at every k -equilibrium. At the limit, however, $z_1^h = K$ and $b^h = K$ for all h , implying $z_1 > 0$ and $\sum_h b^h > 0$. This contradicts the open-graph property of household preferences. Hence, this case is ruled out and π^k must converge to a finite limit, where Definition 4.1 holds, as concluded in the penultimate paragraph. This completes the proof of Theorem 4.2. ■

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