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for action-dependent bargaining protocols

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**On the convergence to the Nash bargaining solution
for action-dependent bargaining protocols**

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Abstract

We consider a non-cooperative multilateral bargaining game and study an action-dependent bargaining protocol, that is, the probability with which a player becomes the proposer in a round of bargaining depends on the identity of the player who previously rejected. An important example is the frequently studied rejector-becomes-proposer protocol. We focus on subgame perfect equilibria in stationary strategies which are shown to exist and to be efficient. Equilibrium proposals do not depend on the probability to propose conditional on the rejection by another player, though equilibrium acceptance sets do depend on these probabilities. Next we consider the limit, as the bargaining friction vanishes. In case no player has a positive probability to propose conditional on his rejection, each player receives his utopia payoff conditional on being recognized and equilibrium payoffs are in general Pareto inefficient. Otherwise, equilibrium proposals of all players converge to a weighted Nash Bargaining Solution, where the weights are determined by the probability to propose conditional on a rejection.

Keywords: strategic bargaining, subgame perfect equilibrium, stationary strategies, Nash bargaining solution.

JEL Classification: C78

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1 Introduction

This paper examines the convergence of equilibrium payoffs to the asymmetric Nash bargaining solution in a non-cooperative bargaining game. In contrast to the existing literature on this topic, we allow for the proposer selection process to be action-dependent, that is, influenced by the players' actions throughout the game.

The study of non-cooperative bargaining games has been strongly influenced by Rubinstein (1982). In his setup, two impatient players take turns in proposing a division of some surplus until one player agrees to the opponent's current offer. The unique division is supported by subgame-perfect equilibrium. The corresponding equilibrium strategies are stationary. The equilibrium payoffs depend on the degree of the players' impatience and converge to the well-known Nash bargaining solution (Nash 1950, 1953) in the limit as the players' impatience vanishes. A similar support result for the Nash bargaining solution is due to Binmore, Rubinstein, and Wolinsky (1986). These results cannot be easily reproduced in setups with more than two players. In particular, the subgame-perfect equilibrium payoffs can no longer be predicted uniquely.¹

It is common in the literature to restrict attention to those subgame-perfect equilibria which are in stationary strategies, an early example is Hart and Mas-Colell (1996). This allows for sharp predictions of equilibrium payoffs in the limit as the cost of delay becomes small. For instance, Kultti and Vartiainen (2010) find convergence to the Nash bargaining solution when Rubinstein's model is extended to an arbitrary number of players who take turns proposing in a fixed order. One alternative model selects the proposer in each round according to a time-invariant probability distribution. Laruelle and Valenciano (2008) and Miyakawa (2008) show the convergence of equilibrium payoffs to an asymmetric Nash bargaining solution (Kalai 1977) where the time-invariant probability distribution corresponds to the vector of bargaining weights.

Both of the above results are special cases of Britz, Herings, and Predtetchinski (2010), who model the proposer selection process as a Markov chain, and obtain convergence to the asymmetric Nash bargaining solution where the weight vector is given by the stationary distribution of the Markov chain. The selection of proposers by a Markov chain, and all its special cases, is action-independent: The actions taken by the players in the game have no effect on the identity of the next proposer. To the best of our knowledge, the entire literature that has provided non-cooperative support for the asymmetric Nash bargaining

¹One way to restore the uniqueness of subgame-perfect equilibrium is to consider a bargaining process where an agreement is reached in several steps and only a subset of the players bargain with each other at each step. Examples of such "partial agreements" can be found in Chae and Yang (1994), Krishna and Serrano (1996), and Suh and Wen (2006). A similar approach has been applied to a coalition formation problem by Moldovanu and Winter (1995).

solution considers action-independent protocols only.

We argue that this is a serious limitation because action-dependent protocols are very common in the rest of the bargaining literature. One simple and intuitively appealing example is the protocol where the player who rejects the current proposal is automatically called to make the next proposal. This rejector-becomes-proposer protocol has been introduced in Selten (1981) and studied extensively in both the bargaining and the coalition formation literature, see for example Chatterjee, Dutta, Ray, and Sengupta (1993), Bloch (1996), Ray and Vohra (1999), Imai and Salonen (2000), and Bloch and Diamantoudi (2011). The protocol we study in this paper is more general than the rejector-becomes-proposer protocol. Following Kawamori (2008), we are interested in the case where the identity of the player who rejects a proposal may influence the probability by which a particular player becomes the next proposer. Since the accept/reject decisions of the players influence the proposer selection, this is indeed an action-dependent protocol. Such protocols are considerably more difficult to analyze than action-independent ones, and the literature has identified a number of cases where both types of protocol lead to surprisingly different results. For instance, Chatterjee, Dutta, Ray, and Sengupta (1993) provide examples for non-existence of equilibria as well as existence of equilibria with delay in the context of an action-dependent protocol. On the contrary, it has been shown in Okada (1996) that delay cannot occur at equilibrium and in Okada (2011) that equilibria exist when the protocol is action-independent.²

Our results shed new light on the influence of the proposer selection process on the bargaining outcome. Both theoretical and experimental research has emphasized the importance of proposal making for bargaining power, see for instance Romer and Rosenthal (1978), Knight (2005), and Kalandrakis (2006).

Some of our main findings are as follows. We first consider all subgame-perfect equilibria in stationary strategies. Equilibrium proposals do not depend on the probability to propose conditional on the rejection by another player. Regarding results on the limit of equilibrium payoffs as the continuation probability tends to one, we find a distinction between two cases.

If none of the players has a positive probability of being the next proposer after his own rejection, then the proposer in the initial round obtains his utopia payoff, that is his highest payoff in the set of feasible payoffs that satisfy all the individual rationality constraints. The equilibrium proposals of all players are independent of the continuation probability

²Similarly, there are examples for non-existence of equilibrium in Bloch (1996) under an action-dependent protocol while Herings and Houba (2010) restore existence for an action-independent proposer selection protocol. Duggan (2011) present a very general coalitional bargaining model where equilibrium existence is shown for action-independent protocols. The paper points out that a similar approach to establish equilibrium existence would not work when the protocol is action-dependent.

and do not converge to a common limit. Since the initial proposer is selected according to some given probability distribution, the utilities are in general not Pareto efficient, and do therefore not correspond to an asymmetric Nash bargaining solution.

Otherwise, we find convergence of the equilibrium payoffs to a weighted Nash bargaining solution, where the weights are determined by the probabilities of making a counter-offer. Players with a zero probability of making a counter-offer receive a payoff of zero. The existing results on non-cooperative bargaining games are for action-independent protocols only and do not distinguish between the probability of making a proposal and the probability of making a proposal conditional on a rejection. Our paper argues that the latter probabilities are the ones that really matter to explain bargaining power.

The paper is organized as follows. Section 2 formally introduces bargaining games with action-dependent protocols. Section 3 presents a characterization of the set of subgame-perfect equilibria in stationary strategies and shows that such equilibria exist. Section 4 discusses the relationship between the payoffs of stationary subgame-perfect equilibria and the asymmetric Nash bargaining solution. In Section 5, we illustrate the findings of Section 4 with an example. Section 6 concludes.

2 The Bargaining Game

We consider a bargaining game between finitely many players. The set of players is $N = \{1, \dots, n\}$. Each player individually can only attain a disagreement payoff which we normalize to zero. However, the players can jointly achieve any payoff vector v in a set $V \subset \mathbb{R}^n$ if they unanimously agree on such a payoff vector. Each player is assumed to be an expected utility maximizer. The set V of feasible payoffs and the bargaining protocol are the main primitives of the model. We now introduce each in turn.

For vectors u and v in \mathbb{R}^n , we write $u \geq v$ if $u_i \geq v_i$ for all $i \in N$, $u > v$ if $u \geq v$ and $u \neq v$, and $u \gg v$ if $u_i > v_i$ for all $i \in N$. A point v of V is said to be *Pareto-efficient* if there is no point u in V such that $u > v$. A point v of V is said to be *weakly Pareto-efficient* if there is no point u in V such that $u \gg v$. We write V_+ to denote the set $V \cap \mathbb{R}_+^n$.

Our first assumption is as follows:

- [A1] The set V is closed, convex, and comprehensive from below. There is a point $v \in V$ such that $v \gg 0$. The set V_+ is bounded. Each weakly Pareto-efficient point of V_+ is Pareto-efficient.

We denote the set of Pareto-efficient points of V by P and write P_+ for the set $P \cap \mathbb{R}_+^n$.

Bargaining takes place in discrete time $t = 0, 1, \dots$. There are $n + 1$ probability distributions on the players denoted by $\pi^0, \pi^1, \dots, \pi^n$, each of which belongs to the unit simplex Δ^n in \mathbb{R}^n .

In round $t = 0$, a particular player is chosen as the proposer according to the probability distribution $\pi^0 \in \Delta^n$. The proposer then makes a proposal $v \in V$. Player 1 responds to the proposal by either acceptance or rejection. Once a player $i = 1, \dots, n-1$ has accepted the proposal, it is the turn of player $i+1$ to accept or reject.³ Once player n has accepted the proposal, the game ends and the approved proposal is implemented.

As soon as some player $j \in N$ rejects a proposal in round t , the game ends with probability $1 - \delta > 0$ and payoffs to all players are zero. With the complementary probability δ , the game continues to round $t+1$. The proposer in that round is then drawn from the probability distribution π^j . If the game continues perpetually without agreement, the payoff to every player is zero.

The rejector-becomes-proposer protocol follows from specifying $\pi_i^i = 1$ for all $i \in N$. A polar opposite of the rejector-becomes-proposer protocol, where a rejector proposes with probability zero in the next round, follows by setting $\pi_i^i = 0$ for all $i \in N$. In case $\pi^0, \pi^1, \dots, \pi^n$ all coincide, we are back in the familiar case of an action-independent protocol with time-invariant recognition probabilities.

It is well-known that bargaining games with more than two players admit a wide multiplicity of subgame-perfect equilibria (SPE), see Herrero (1985) and Haller (1986). We will restrict attention to subgame-perfect equilibria in stationary strategies (SSPE). A *stationary strategy* for player i consists of a proposal $\theta^i \in V$ which i makes whenever it is his turn to propose and an acceptance set $A^i \subset V$ which consists of all the proposals which player i would be willing to accept if they were offered to him. We denote the *social acceptance set* by $A = \cap_{i \in N} A^i$ and write the profile of stationary strategies $(\theta^1, A^1, \dots, \theta^n, A^n)$ more concisely as (Θ, \mathcal{A}) .

3 Subgame Perfect Equilibria in Stationary Strategies

In this section, we consider the set of SSPEs of the bargaining game. Fix some profile of stationary strategies (Θ, \mathcal{A}) . By definition of a stationary strategy, there is a unique payoff vector which is expected in any subgame following a rejection by some player $i \in N$. We refer to it as the vector of *continuation payoffs* after i 's rejection and denote it by $q^i(\Theta, \mathcal{A})$. Since it will be clear from the context, we omit the argument in the sequel. Moreover, we define a vector $r(\Theta, \mathcal{A})$ of *reservation payoffs* by $r(\Theta, \mathcal{A}) = (q_1^1, \dots, q_n^n)$. Again, we will omit the argument in the sequel. Under an action-independent protocol, one and the same vector of continuation payoffs would result no matter which player rejected the current proposal. Consequently, the reservation and continuation payoff vectors would

³Throughout the paper, for the sake of simplicity, we assume that players respond to a proposal in the fixed order $1, \dots, n$. All results would carry over to the case with arbitrary voting orders.

all be equal to each other. If, however, we allow for an action-dependent protocol, the reservation payoff vector is not generally equal to any of the continuation payoff vectors. This disparity between the reservation and continuation payoffs complicates the SSPE analysis somewhat compared to action-independent protocols.

We will see that the reservation payoffs are important as an “acceptance threshold” in a sense to be made precise in the next lemma.

For every $i \in N$, let $S(i) = \{j \in N \mid j \geq i\}$. That is, $S(i)$ is the set of players succeeding player i in the response order, including i himself. We denote $\cap_{j \in S(i)} A^j$ by $A^{S(i)}$.

Lemma 3.1 *Let (Θ, \mathcal{A}) be an SSPE inducing reservation payoffs r . It holds that*

1. *If $v \in V$ is such that $v_n > r_n$, then $v \in A^n$.*
2. *For every $i = 1, \dots, n-1$, if $v \in A^{S(i+1)}$ and $v_i > r_i$, then $v \in A^i$.*
3. *For every $i = 1, \dots, n$, if $v \in A^{S(i)}$, then $v_j \geq r_j$ for all $j \in S(i)$.*

Proof: Consider a history of the game where player n has to respond to the proposal v with $v_n > r_n$. If player n accepts, the proposal passes and he receives v_n . If he rejects, he expects to receive r_n , so he would have a profitable deviation at this history. This establishes the first part of the lemma. Now consider a history of the game where player $i \in N \setminus \{n\}$ has to respond to the proposal v with $v_i > r_i$ and $v \in A^{S(i+1)}$. If player i accepts v , this proposal passes and he receives v_i . Otherwise, he receives r_i , so he would have a profitable deviation at this history. It remains to show the third part of the lemma. Suppose by way of contradiction that $v \in A^{S(i)}$ but there is $j \geq i$ such that $v_j < r_j$. If player j unilaterally deviates to reject rather than accept the proposal v , he receives a payoff of r_j . This deviation is profitable. \square

Lemma 3.1 describes the role of the vector r as an acceptance threshold in an SSPE. For a payoff vector to be accepted, it needs to be weakly greater than r in all components. Conversely, being strictly greater than r in all components suffices for acceptance. Under an action-independent protocol, it would be immediate that the vector of reservation payoffs belongs to the feasible set since it is equal to the continuation payoffs. With an action-dependent protocol, however, we have to show explicitly that the reservation payoff vector belongs to the feasible set and that it is therefore possible for a proposer to make a proposal which is unanimously acceptable. This is the claim of the next lemma.

Lemma 3.2 *Let (Θ, \mathcal{A}) be an SSPE inducing reservation payoffs r . Then there exists $v \in V$ such that $v \gg r \geq 0$. In particular, it holds that $v \in A$ and $r \in V_+$.*

Proof: Any player i can choose to reject all proposals, a strategy that never leads to an agreement irrespective of the strategy used by the other players, and a payoff of zero for all players. It follows that player i 's payoff in any subgame perfect equilibrium cannot be smaller than zero. In particular, it follows that $r \geq 0$.

Suppose now that there is no $v \in V$ such that $v \gg r$. In view of Assumption A1, there is no $v \in V$ such that $v > r$. It now follows from Lemma 3.1.3 that $A \subset \{r\}$. First suppose that $A = \emptyset$. In this case equilibrium strategies lead to payoffs of zero for all players, so $r = 0$. But under Assumption A1 there is a vector $v \in V$ with $v \gg 0$, a contradiction to our supposition. Hence $A = \{r\}$.

Then, after a rejection, only two outcomes are possible: Either agreement on r is reached at some future time or zero payoffs result. The vector of players' continuation payoffs after a rejection is therefore a convex combination of 0 and r , where the former has a weight of at least $1 - \delta$. But this implies $r_i \leq \delta r_i$ for all $i \in N$. Since $\delta < 1$, we conclude that $r = 0$. As before, this leads to a contradiction.

We conclude that there is a $v \in V$ such that $v \gg r$. Parts 1 and 2 of Lemma 3.1 imply that $v \in A$. The fact that V is comprehensive from below implies that $r \in V$. \square

Lemma 3.2 implies that at an SSPE a proposer is always able to make a proposal which gives all players a strictly higher payoff than their reservation payoffs, and which will therefore be accepted. In a model with an action-independent protocol such as Britz, Herings, and Predtetchinski (2010), it would be immediate that the proposer finds it in his best interest to make such an acceptable proposal. Under an action-dependent protocol, however, the aforementioned disparity between the continuation and reservation payoffs leads to a complication here. In particular, if some continuation payoff q_i^j is sufficiently high, one might conjecture that player i could obtain a higher payoff by making a proposal which will be rejected by player j than by making an acceptable proposal himself. The next step in our argument is to show that no such behavior is consistent with SSPE. On the contrary, immediate agreement is reached in an SSPE.

Lemma 3.3 *Let (Θ, \mathcal{A}) be an SSPE. For all $i \in N$ it holds that $\theta^i \in A$ and $\theta_i^i > 0$.*

Proof: Let u_i be the SSPE utility to player i at a history where it is player i 's turn to make a proposal. It holds that $u_i = \theta_i^i$ if $\theta^i \in A$ and $u_i = q_i^j$ if $\theta^i \notin A$, where j is the least element of N such that $\theta^i \notin A^j$.

By making a proposal $v \in A$, player i guarantees himself a payoff of v_i . It follows that $u_i \geq v_i$ for every $v \in A$. In particular $u_i > 0$ since by Lemma 3.2 there is a vector $v \in A$ such that $v_i > 0$.

Let $U = \{0\} \cup (A \cap \{\theta^1, \dots, \theta^n\})$. This is the set of all possible outcomes of the game if play follows the strategy (Θ, \mathcal{A}) . Take any $j \in N$. The vector q^j of continuation payoffs

is a convex combination of the vectors in U , with 0 having a weight of at least $1 - \delta$. We know from the preceding paragraph that $u_i \geq \theta_i^k$ for all $\theta^k \in U$ and that $u_i > 0$. It follows that $u_i > q_i^j$. Since this holds for each $j \in N$, we have $\theta^i \in A$. Moreover, $\theta_i^i = u_i > 0$. \square

The following lemma claims that SSPE proposals are Pareto-efficient and give each responding player exactly their reservation payoffs. The proof is standard in the bargaining literature and is therefore omitted.

Lemma 3.4 *Let (Θ, \mathcal{A}) be an SSPE inducing reservation payoffs r . For every $i \in N$, it holds that $\theta^i \in P_+$ and $\theta_j^i = r_j$ for all $j \in N \setminus \{i\}$.*

For every $i \in N$, we define $\alpha_i \in [0, 1)$ by

$$\alpha_i = \frac{\delta \pi_i^i}{1 - \delta + \delta \pi_i^i}.$$

Since each proposal belongs to the social acceptance set, the reservation payoffs can be computed as follows:

$$r_i = \delta \sum_{j=1}^n \pi_j^i \theta_j^i = \delta \pi_i^i \theta_i^i + \delta(1 - \pi_i^i) r_i.$$

Solving for r_i , we see that

$$r_i = \alpha_i \theta_i^i.$$

Theorem 3.5 below collects all the necessary conditions that we have derived so far for a strategy profile to be an SSPE.

Theorem 3.5 *If (Θ, \mathcal{A}) is an SSPE inducing reservation payoffs r , then*

$$A^{S(i)} \subset \cap_{j=i, \dots, n} \{v \in V | v_j \geq r_j\}, \quad i \in N, \quad (3.1)$$

$$A^n \supset \{v \in V | v_n > r_n\}, \quad (3.2)$$

$$A^i \supset \{v \in A^{S(i+1)} | v_i > r_i\}, \quad i \in N \setminus \{n\}, \quad (3.3)$$

$$\theta_j^i = r_j, \quad i \in N, \quad j \in N \setminus \{i\}, \quad (3.4)$$

$$r_i = \alpha_i \theta_i^i, \quad i \in N, \quad (3.5)$$

$$\theta^i \in P_+ \cap A, \quad i \in N. \quad (3.6)$$

Conversely, we now turn to sufficient conditions for an SSPE. Consider the following system of equations.

$$\theta^i \in P_+, \quad (3.7)$$

$$\theta_j^i = \alpha_j \theta_j^j, \quad j \in N \setminus \{i\}. \quad (3.8)$$

Theorem 3.6 *Let the proposals Θ satisfy Equations (3.7)–(3.8). There exists a collection of sets $\mathcal{A} = (A^1, \dots, A^n)$ such that (Θ, \mathcal{A}) is a stationary subgame perfect equilibrium.*

Proof: Let Θ be proposals satisfying Equations (3.7) and (3.8). We define acceptance sets A^1, \dots, A^n inductively as follows: Let

$$A^n = \{v \in V \mid v_n \geq \alpha_n \theta_n^n\}.$$

Now suppose that for some player $i \in N \setminus \{n\}$, the acceptance set A^j has been defined for all $j \in S(i+1)$. For any $v \in V$, we define player i 's *payoff upon acceptance of v* as follows:

$$\beta_i(v) = \begin{cases} v_i, & \text{if } v \in \bigcap_{j=i+1}^n A^j, \\ \delta \theta_i^i \left(\frac{\pi_i^k - \delta \pi_i^k + \delta \pi_i^i}{1 - \delta + \delta \pi_i^i} \right), & \text{if } v \notin \bigcap_{j=i+1}^n A^j, \quad k = \min\{j \geq i+1 : v \notin A^j\}. \end{cases}$$

As will become clear from the subsequent discussion, the payoff $\beta_i(v)$ corresponds to the payoff to player i from accepting the proposal v . We define

$$A^i = \{v \in V \mid \beta_i(v) \geq \alpha_i \theta_i^i\}.$$

The construction of acceptance sets \mathcal{A} is now complete. We denote by A the concomitant social acceptance set $A^1 \cap \dots \cap A^n$.

We claim that $\theta^i \in A$ for each i . Indeed, by Equation (3.8) and because $\alpha_j \leq 1$ it holds that $\theta_j^i \geq \alpha_j \theta_j^j$ for each i and j . Now we show that $\theta^i \in A^j$ for every j by induction on j . Since $\theta_n^i \geq \alpha_n \theta_n^n$ we have $\theta^i \in A^n$. Now suppose that for some $j \in N \setminus \{n\}$ it holds that $\theta^i \in A^k$ for all $k \in S(j+1)$. Then $\beta_j(\theta^i) = \theta_j^i \geq \alpha_j \theta_j^j$. It follows that $\theta^i \in A^j$, thereby completing the induction step.

Since player j 's proposal θ^j is accepted, the continuation payoff to player i after a rejection of a proposal by player k can now be computed as follows:

$$q_i^k = \delta \sum_{j=1}^n \pi_j^k \theta_j^i = \delta \pi_i^k \theta_i^i + \delta (1 - \pi_i^k) \alpha_i \theta_i^i = \delta \theta_i^i \left(\frac{\pi_i^k - \delta \pi_i^k + \delta \pi_i^i}{1 - \delta + \delta \pi_i^i} \right). \quad (3.9)$$

The reservation payoffs are given by

$$r_i = q_i^i = \alpha_i \theta_i^i.$$

The payoff $\beta_i(v)$ upon acceptance of v is v_i if $v \in A^j$ for every player $j \in S(i+1)$ and it is q_i^k otherwise, where k is the lowest indexed player in $S(i+1)$ with $v \notin A^k$. Thus $\beta_i(v)$ is the payoff to i from accepting the proposal v . Under the strategy (Θ, \mathcal{A}) , player i accepts the proposal v if and only if $\beta_i(v) \geq r_i$.

We claim that (Θ, \mathcal{A}) is an SSPE. Clearly, it is a profile of stationary strategies. To show that it is an SPE, applying the one-shot deviation property, we have to show that at any history the acting player has no profitable one-shot deviation.

Consider first a one-shot deviation in the accept/reject decisions. The preceding discussion shows that a responder accepts a proposal if and only if accepting makes him weakly better off than rejecting. Hence, a one-shot deviation cannot be profitable.

Now consider a one-shot deviation by player i , who proposes some $v \in V \setminus \{\theta^i\}$. Suppose that $v \notin A$. Then, the expected payoff to player i from proposing v is q_i^k where k is the first player in the response order to reject v . It follows from Equation (3.9) that $q_i^k \leq \delta \theta_i^i \leq \theta_i^i$, so the deviation is not profitable. Now consider the case where $v \in A$. Then $v_j \geq \alpha_j \theta_j^j$ for every $j \in N$ and hence $v_j \geq \theta_j^i$ for every $j \in N \setminus \{i\}$. Since the vector θ^i is Pareto-efficient, we must have $v_i \leq \theta_i^i$. Again, the deviation is not profitable. \square

Theorem 3.7 *There exist proposals Θ which solve Equations (3.7) and (3.8).*

Proof: Let us define $\rho \in \Delta^n$ and $\lambda \in [0, 1)$ as follows. If $\pi_i^i > 0$ for at least one $i \in N$, then

$$\begin{aligned} \rho_i &= \frac{\pi_i^i}{\sum_{j=1}^n \pi_j^j}, & i \in N, \\ \lambda &= \frac{\delta \sum_{i=1}^n \pi_i^i}{1 - \delta + \delta \sum_{i=1}^n \pi_i^i}. \end{aligned}$$

If $\pi_i^i = 0$ for all $i \in N$, then we set $\lambda = 0$ and $\rho_i = \frac{1}{n}$ for all $i \in N$. After elementary manipulations, Equations (3.7) and (3.8) reduce to the system of characteristic equations which describes equilibrium proposals in a bargaining model with time-invariant recognition probabilities ρ and continuation probability λ . The existence of a solution to the latter system has been shown by Banks and Duggan (2000) in their Theorems 1 and 2.⁴ \square

4 SSPE payoffs and the Nash bargaining solution

In bargaining games with action-independent protocols, all SSPE proposals converge to a common limit as the continuation probability goes to one. Since all proposals are Pareto-efficient for all values of the continuation probability, the common limit of SSPE proposals is Pareto-efficient as well. It corresponds to an asymmetric Nash bargaining solution, where

⁴In Banks and Duggan (2000) the continuation probability is 1, and players have time preferences with a discount factor equal to δ . Voting is simultaneous rather than sequential, and attention is restricted to stage-undominated voting strategies. For the case with time-invariant recognition probabilities, both modeling choices lead to exactly the equilibrium conditions on proposals given by (3.7) and (3.8). The equilibrium conditions are also the same as in a rejector-becomes-proposer model where players have heterogeneous time discount factors α_i .

the bargaining weights are given by the transition probabilities involved in the bargaining protocol. Under an action-dependent protocol, however, it is not always true that all SSPE proposals converge to the same limit. In particular, SSPE proposals fail to converge when $\pi_i^i = 0$ for all $i \in N$. We deal with this case in the first subsection below, and we address the case where $\pi_i^i > 0$ for at least one $i \in N$ in the second subsection below.

4.1 Failure of Convergence

One important implication of the foregoing analysis is that a responding player who does not have positive probability of making a counter-offer will not be offered a positive payoff. Indeed, if $\pi_i^i = 0$ for some player i , then $\alpha_i = 0$ and so $r_i = 0$.

Lemma 4.1 *Let (Θ, \mathcal{A}) be an SSPE inducing reservation payoffs r and let $i \in N$ be a player with $\pi_i^i = 0$. Then it holds that $r_i = 0$ and $\theta_j^i = 0$ for all $j \in N \setminus \{i\}$.*

We define \hat{v} as the vector of *utopia payoffs*, where the utopia payoff of a player $i \in N$ is the highest payoff in V for player i that satisfies all the individual rationality constraints, that is $\hat{v}_i = \max\{v_i \in \mathbb{R} \mid v \in V_+\}$. Consider the case where *all* players have zero probability to make a counter-offer conditional on a rejection. That is, suppose $\pi_i^i = 0$ for all $i \in N$. In that case, Theorem 3.5 readily implies that at any history where player i is the proposer, he receives his utopia payoff in an SSPE.

Theorem 4.2 *Assume $\pi_i^i = 0$ for all $i \in N$. In any SSPE, the payoff to player $i \in N$ is equal to $\pi_i^0 \hat{v}_i$.*

It is important to note that the above statement does not involve the discount factor δ at all. In the case where $\pi_i^i = 0$ for all $i \in N$ the initial proposer always receives his utopia payoff. The proposals involved in an SSPE do not converge to a common limit. If P_+ is not a simplex and if $\pi^0 \gg 0$, this implies in addition that the ex ante SSPE payoffs allocation is Pareto-inefficient and does not correspond to any asymmetric Nash bargaining solution. There is a feasible payoff allocation which all players would prefer over starting the bargaining game and playing the strategies prescribed by an SSPE.

4.2 Convergence to a Nash bargaining solution

In this section, we deal with the case where at least one player does have a positive probability to make the next proposal following his own rejection. From now on we impose Assumptions A2 and A3 below. A vector η in \mathbb{R}^n is a normal vector to V at a point $v \in V$ if $(u - v)^\top \eta \leq 0$ for all $u \in V$. It is said to be a unit normal vector if $\|\eta\| = 1$.

[A2] There is $i \in N$ such that $\pi_i^i > 0$.

[A3] There is a continuous function $\eta : P_+ \rightarrow \mathbb{R}^n$ such that $\eta(v)$ is a unit normal vector to V at the point v .

Assumption A3 implies that the boundary P_+ does not have kinks. Notice that in view of Assumption A1 we have $\eta_i(v) > 0$ for every $i \in N$ such that $v_i > 0$.

All the existing results on multilateral bargaining that show the convergence of equilibrium payoffs to an asymmetric Nash bargaining solution under an action-independent protocol rely on Assumption A3. Indeed, without such an assumption, Kultti and Vartiainen (2010) provide an example where SSPE payoffs fail to converge to the Nash bargaining solution and Herings and Predtetchinski (2011) show that the limit of SSPE payoffs may not be unique.

Lemma 4.3 *For every $m \in \mathbb{N}$, let Θ_m be SSPE equilibrium proposals of the game with continuation probability δ_m . Suppose that the sequence $(\delta_m)_{m \in \mathbb{N}}$ converges to 1. If the sequence $(\Theta_m)_{m \in \mathbb{N}}$ converges to $\bar{\Theta}$, then $\bar{\theta}^1 = \dots = \bar{\theta}^n$. Moreover, for every $i \in N$, the point $\bar{\theta}^i$ is Pareto-efficient.*

Proof: We prove first that for every $i \in N$ the point $\bar{\theta}^i$ is Pareto-efficient. Suppose not. Then, in view of Assumption A1, there is $i \in N$ and $v \in V$ such that $v \gg \bar{\theta}^i$. But then $v \gg \theta_m^i$ for m large enough, which contradicts the fact that $\theta_m^i \in P_+$.

Take $i \in N$ such that $\pi_i^i > 0$ and take any other player $j \in N$. We wish to show that $\bar{\theta}^j = \bar{\theta}^i$. For every $m \in \mathbb{N}$, for every $k \in N \setminus \{i, j\}$, it holds that $\theta_{m,k}^i = \theta_{m,k}^j = \alpha_{m,k} \theta_{m,k}^k$. Taking the limit as m goes to infinity yields $\bar{\theta}_k^i = \bar{\theta}_k^j$. Furthermore, for every $m \in \mathbb{N}$, we have $\theta_{m,i}^j = \alpha_{m,i} \theta_{m,i}^i$. Since $\alpha_{m,i}$ converges to one as m tends to infinity, we obtain $\bar{\theta}_i^j = \bar{\theta}_i^i$. We conclude that the vectors $\bar{\theta}^i$ and $\bar{\theta}^j$ can only differ in component j . Since both points $\bar{\theta}^i$ and $\bar{\theta}^j$ are Pareto-efficient, we have $\bar{\theta}^i = \bar{\theta}^j$, as desired. \square

The lemma above therefore justifies the following definition.

Definition 4.4 *A limit equilibrium proposal is a proposal $\theta \in V$ for which there exist sequences $(\delta_m)_{m \in \mathbb{N}}$ and $(\Theta_m)_{m \in \mathbb{N}}$, where Θ_m are SSPE proposals in the game with continuation probability δ_m , such that $(\theta_m^i)_{m \in \mathbb{N}}$ converges to $\bar{\theta}$ for every $i \in N$.*

Since the proposals Θ_m in Lemma 4.3 all belong to the compact set V_+ , every sequence $(\Theta_m)_{m \in \mathbb{N}}$ has a convergent subsequence. This demonstrates the existence of a limit equilibrium proposal. Lemma 4.3 implies that it is efficient.

Corollary 4.5 *A limit equilibrium proposal exists. Each limit equilibrium proposal is Pareto efficient.*

Theorem 4.6 *Let $\bar{\theta}$ be a limit equilibrium proposal.*

1. *If $\pi_i^i = 0$ for some $i \in N$, then $\bar{\theta}_i = 0$.*
2. *Suppose there is exactly one player $i \in N$ with $\pi_i^i > 0$. Then $\bar{\theta}_i$ is player i 's utopia payoff \hat{v}_i and $\bar{\theta}_j = 0$ for $j \neq i$.*

Proof: To prove Theorem 4.6.1, take a player $i \in N$ with $\pi_i^i = 0$ and observe that in each SSPE $\theta_i^j = 0$ for every $j \neq i$. Taking the limit of $\theta_{m,i}^j$ along the appropriate sequence of SSPEs yields $\bar{\theta}_i = 0$. To prove Theorem 4.6.2, notice that $\bar{\theta}_j = 0$ for $j \neq i$ by Theorem 4.6.1. The fact that $\bar{\theta}_i$ is player i 's utopia payoff follows from the fact that $\bar{\theta}$ is a Pareto-efficient point of V . \square

We proceed by showing that the limit equilibrium proposal is unique and is equal to the asymmetric Nash bargaining solution where player i has weight π_i^i . Given a vector $\lambda \in \mathbb{R}_+^n \setminus \{0\}$, we define the λ -Nash product $\rho_\lambda : \mathbb{R}_+^n \rightarrow \mathbb{R}$ by

$$\rho_\lambda(v) = \prod_{i \in N} v_i^{\lambda_i}, \quad v \in \mathbb{R}_+^n.$$

Definition 4.7 *Given a vector $\lambda \in \mathbb{R}_+^n \setminus \{0\}$, the maximizer of the function ρ_λ on V_+ is called the λ -Nash bargaining solution.*

Under our assumptions the maximizer of the function ρ_λ on V_+ is indeed unique. It is a Pareto-efficient point of V which is uniquely characterized by the following conditions: for each i and j in N ,

$$\text{if } \lambda_i = 0, \text{ then } v_i = 0, \tag{4.1}$$

$$\text{if } \lambda_i, \lambda_j > 0, \text{ then } \frac{v_i \eta_i(v)}{\lambda_i} = \frac{v_j \eta_j(v)}{\lambda_j}. \tag{4.2}$$

Theorem 4.8 *The limit equilibrium proposal is unique and is equal to the $(\pi_1^1, \dots, \pi_n^n)$ -Nash bargaining solution.*

Proof: We verify that each limit equilibrium proposal satisfies the conditions (4.1)–(4.2) with $\lambda_i = \pi_i^i$. Let

$$\tilde{N} = \{i \in N \mid \pi_i^i > 0\}.$$

Let $(\theta^1, \dots, \theta^n)$ be SSPE proposals in a game with continuation probability δ . By the definition of the normal vector it holds for any two players i and j that

$$(\theta^j - \theta^i)^\top \eta(\theta^i) \leq 0.$$

Notice that the proposals θ^i and θ^j can only differ in components i and j . Solving for the inner product, we can therefore rewrite the previous inequality as

$$(\theta_j^j - \theta_j^i)\eta_j(\theta^i) + (\theta_i^j - \theta_i^i)\eta_i(\theta^i) \leq 0.$$

Substituting for θ_j^i and θ_j^j from equation (3.8) and dividing by $1 - \delta$ yields

$$\frac{\theta_j^j \eta_j(\theta^i)}{1 - \delta + \delta \pi_j^j} \leq \frac{\theta_i^i \eta_i(\theta^i)}{1 - \delta + \delta \pi_i^i}.$$

Let $\bar{\theta}$ be a limit equilibrium proposal. Taking the limit of the latter inequality along a sequence of equilibrium proposals converging to $\bar{\theta}$ we obtain for all $i, j \in \tilde{N}$,

$$\frac{\bar{\theta}_j \eta_j(\bar{\theta})}{\pi_j^j} \leq \frac{\bar{\theta}_i \eta_i(\bar{\theta})}{\pi_i^i}.$$

Interchanging the roles of the players i and j , we obtain the equality

$$\frac{\bar{\theta}_j \eta_j(\bar{\theta})}{\pi_j^j} = \frac{\bar{\theta}_i \eta_i(\bar{\theta})}{\pi_i^i}, \quad i, j \in \tilde{N}. \quad (4.3)$$

This shows that $\bar{\theta}$ satisfies (4.2). The fact that $\bar{\theta}$ satisfies (4.1) follows at once from Corollary 4.6.1. \square

5 Example

The analysis of SSPE under an action-dependent protocol reveals two rather striking features. First, only the “diagonal” probabilities π_1^1, \dots, π_n^n are relevant for the SSPE payoffs, together with the initial probability to propose π^0 in case $\pi_1^1 = \dots = \pi_n^n = 0$. The chance to become a proposer after a rejection by another player is not a source of bargaining power. A player whose probability to become a proposer after his own rejection is zero, will never be offered a positive payoff. Second, the SSPE payoffs exhibit a discontinuity when all the probabilities π_1^1, \dots, π_n^n are equal to zero. In that case, whoever is the proposer in the initial round can appropriate the entire surplus, which is no longer the case when π_i^i is non-zero for at least one $i \in N$.

The following example illustrates these two features. There are four players $N = \{1, 2, 3, 4\}$. The vectors π^1, \dots, π^4 are given by

$$(\pi^1, \pi^2, \pi^3, \pi^4) = \begin{pmatrix} 0 & 1 & 1 & 1 - \varepsilon \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \varepsilon \end{pmatrix}.$$

We first assume that $\varepsilon > 0$ and suppose that player 4 is the initial proposer with probability zero. Player 4 appears very weak in the example at hand. In order for player 4 to become the proposer at all, some proposal must be accepted by players 1, 2, and 3 and then be rejected by player 4. Even in that case, player 4 becomes the next proposer only with probability ε , which can be arbitrarily close to zero. Player 1 on the other hand, could be the initial proposer with probability as high as one, becomes the next proposer for sure whenever player 2 or player 3 makes a rejection, and becomes the next proposer with probability $1 - \varepsilon$ whenever player 4 rejects. Nevertheless, our convergence result implies that player 4 has all the bargaining power and receives his utopia payoff under the limit equilibrium proposal. On the contrary, players 1, 2, and 3 receive zero payoffs in the limit equilibrium proposal.

Assume next that $\varepsilon = 0$. In that case, we no longer have the convergence result presented in Section 4.2, but rather, we know from Section 4.1 that the initial distribution π^0 determines the equilibrium payoffs, which, moreover, are independent from the continuation probability δ . Since we assumed that player 4 is the initial proposer with probability zero, he now receives an equilibrium payoff of zero. In the example at hand, player 3 appears to be a “weak” player in the sense that he can never become a proposer except in the very first round. One would expect that if δ is large enough, player 3’s bargaining power vanishes. However, irrespective of the value of δ , player 3 obtains his utopia payoff in equilibrium when he is selected as the initial proposer.

6 Conclusion

We have considered multilateral bargaining games with action-dependent protocols. The identity of the player who rejects the current proposal determines the probability distribution from which the next proposer is drawn. Surprisingly, the probability with which a player proposes after another player’s rejection turns out to be irrelevant for the prediction of equilibrium proposals and payoffs. The probability with which a player proposes after his own rejection is crucial for the equilibrium prediction. Our main results highlight a discrepancy between the case where, conditional on his rejection, the probability to make a counter-offer is zero for every player, and the case where for some player this probability is positive.

If the probability to make a counter-offer is zero for all players, then the equilibrium payoffs are determined by the utopia point and the recognition probabilities in the initial round. The resulting equilibrium utilities do not generally converge to an asymmetric Nash bargaining solution. In the case where at least one player does have a strictly positive probability to make a counter-offer, we obtain the convergence to an asymmetric Nash

bargaining solution, where the weights are assigned to the players in the proportion of their probabilities to make a counter-offer. In particular, if all players have the same strictly positive probability to make a counter-offer, then the equilibrium utilities converge to the symmetric Nash bargaining solution, irrespective of how large that probability actually is.

One rather surprising outcome of our analysis is that the probability to become proposer conditional on the rejection of another player does not affect equilibrium payoffs at all. This also sheds new light on the findings in Miyakawa (2008) and Laruelle and Valenciano (2008) concerning the protocol with time-invariant recognition probabilities, which is a special case of our model. Under time-invariant recognition probabilities, it is impossible to discern the effect of the probability to become proposer after one's own rejection as opposed to the probability to propose after another player's rejection. Our more general setup makes the importance of this distinction apparent.

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