UCLouvain

lmat1331

2018

Commutative algebra

4 credits	45.0 h	Q2

Teacher(s)	Tignol Jean-Pierre ;
Language :	French
Place of the course	Louvain-la-Neuve
Prerequisites	Cours LMAT1131. Topics discussed in the course : solution of systems of algebraic equations. Arithmetic of polynomial rings and elimination theory. Structure of modules over a principal ideal domain, and application to the classification of linear operators on finite-dimensional vector spaces. *The prerequisite(s) for this Teaching Unit (Unité d'enseignement – UE) for the programmes/courses that offer this Teaching Unit are specified at the end of this sheet.*
Main themes	Solution of systems of algebraic equations. Arithmetic of polynomial rings and elimination theory. Structure of modules over a principal ideal domain, and application to the classification of linear operators on finite-dimensional vector spaces.
Aims	Contribution of the course to learning outcomes in the Bachelor in Mathematics programme. By the end of this activity, students will have made progress in: - Recognising and understanding a basic foundation of mathematics. -- Choosing and using the basic tools of calculation to solve mathematical problems. -- Recognising the fundamental concepts of important current mathematical theories. -- Establishing the main connections between these theories, analysing them and explaining them through the use of examples. - Identifying, by use of the abstract and experimental approach specific to the exact sciences, the unifying features of different situations and experiments in mathematics or in closely related fields (probability and statistics, physics, computing). - Showing evidence of abstract thinking and of a critical spirit : 1 -- Arguing within the context of the axiomatic method, recognising the key arguments and the structure of a proof, constructing and drawing up a proof independently. -- Evaluating the rigour of a mathematical or logical argument and identifying any possible flaws in it. -- Distinguishing between the intuition and the validity of a result and the different levels of rigorous understanding of this same result. Learning outcomes specific to the course. By the end of this activity, students will be able to: - Factor multivariate polynomials into irreducible factors. - Analyse systems of algebraic equations to determine whether they admit solutions and to represent them geometrically. - Find equations with a given parametrised set of solutions. - Analyse the structure of modules over a principal ideal domain. - Reduce linear operators over a finite-dimensional vector space to canonical forms. - - - - *The contribution of this Teaching Unit to the development and command of the skills and learning outcomes of the programme(s) can be accessed at the end of this sheet, in the section entitled "Programmes/courses offering this Teaching Unit".*
Evaluation methods	Assessment is based on a written examination that focuses on theory and on exercises in more or less equal parts. The examination tests knowledge and understanding of fundamental concepts and results, ability to construct and write a coherent argument, and competence in calculation techniques.
Teaching methods	Learning activities consist of lectures including supervised exercise sessions, some of which are held in the computer laboratory. The lectures aim to introduce fundamental concepts, to explain them by showing examples and by delineating their use, to show their reciprocal connections and their connections with other courses in the programme for the Bachelor in Mathematics. The supervised exercise sessions aim to teach how to select and use calculation methods and how to construct proofs. The two activities are given in presential sessions.
Content	This course introduces abstract algebraic notions related to divisibility, which play an important role throughout the cursus of Bachelor and Master in Mathematics : ideals and factorization in commutative rings, and modules over principal ideal domains. The emphasis lies on polynomial rings, for which many algebraic statements can be illustrated geometrically and established algorithmically. The following topics are discussed :

	- Polynomials and affine algebraic varieties. - Groebner bases of ideals in polynomial rings. - Unique factorisation, resultants, and elimination in polynomial rings. - Existence of solutions for systems of algebraic equations (Hilbert's Nullstellensatz). - Reduction of matrices over principal ideal domains. - Structure of modules of finite type over principal ideal domains : invariant factors and elementary divisors. - Reduction of linear operators to the Jordan or rational canonical form, minimal polynomial of linear operators.
Inline resources	iCampus website (http://icampus.uclouvain.be/). Available on the site are the problems to be solved with the help of computer algebra systems at problem sessions, and problems from examinations of previous years.
Bibliography	Cox, David; Little, John; O'Shea, Donal: "Ideals, varieties, and algorithms. An introduction to computational algebraic geometry and commutative algebra." Third edition. Undergraduate Texts in Mathematics. Springer, New York, 2007. xvi+551 pp. ISBN: 978-0-387-35650-1; 0-387-35650-9
Faculty or entity in charge	MATH

Programmes containing this learning unit (UE)

Program title	Acronym	Credits	Prerequisite	Aims
Bachelor in Mathematics	MATH1BA	4	LMAT1131	
Minor in Mathematics	LMATH100I	4		
Additionnal module in Mathematics	LMATH100P	4		