UCLouvain

lmat1231

2023

Multilinear algebra and group theory

5.00 credits	30.0 h + 30.0 h	Q1

Teacher(s)	Caprace Pierre-Emmanuel ;
Language :	English > French-friendly
Place of the course	Louvain-la-Neuve
Prerequisites	Successful completion of the LMAT1131 exam.
Main themes	Elements of group theory : quotient group and isomorphism theorems, abelianization, cyclic groups, symmetric groups, actions of groups. Multilinear algebra : duality, quotient space, tensor product of vector spaces.
Learning outcomes	**At the end of this learning unit, the student is able to :** **Contribution of the course to learning outcomes in the Bachelor in Mathematics programme.** **By the end of this activity, students will have made progress in :** Know and understand a fundamental foundation of mathematics. In particular, he/she will have developed the ability to: -- Select and use fundamental computational methods and tools to solve mathematical problems. -- Recognize the fundamental concepts of some current mathematical theories. -- Establish the main links between these theories, explain them and motivate them with examples. - To identify, through the abstract and experimental approach of the exact sciences, the unifying aspects of different situations and experiences in mathematics. - Demonstrate abstraction and critical thinking. In particular, he/she will have developed the ability to : -- Reason within the framework of the axiomatic method. -- Recognize the key arguments and the structure of a demonstration. -- Construct and write a demonstration in an autonomous way. -- Appreciate the rigor of a mathematical reasoning and detect possible flaws. 1 -- distinguish between the intuition of the validity of a result and the different levels of rigorous understanding of the same result. **Course-specific learning outcomes.** **At the end of this activity, the student will be able to :** - demonstrate some basic results of group theory; - use some criteria to establish whether a group has one of the properties seen in the course (e.g., being abelian, cyclic, simple, symmetric, etc.); - demonstrate the stability properties of a certain type of group with respect to a given construction (stability by direct products, subgroups, quotients); - recognize the universal properties of algebraic structures and use them to determine whether two structures are isomorphic; - define and study quotients of algebraic structures (groups and vector spaces), analyzing them in concrete examples; - determine if an endomorphism is triangularizable, and in this case find bases of the vector space allowing to triangularize it; - use tensor products to solve multilinear algebra problems.
Evaluation methods	Assessment is based on a written examination during the exam session, covering both theory and exercises, and on personal projects realized during the semester. The examination tests knowledge and understanding of fundamental concepts and results, ability to construct and write a coherent argument, and mastery of the techniques of calculation. Students can choose to write their exams and projects in French or in English.
Teaching methods	Learning activities consist of lectures and exercise sessions. The lectures aim to introduce fundamental concepts, to explain them by showing examples and by supplying complete and detailed proofs of the main results. The exercise sessions are fundamental in apprehending the theoretic content and applying it in solving various problems and realizing simple proofs in an independent way.

Content	In this course some abstract algebraic notions are introduced, which have an essential role in the bachelor and master's courses in mathematical sciences: groups, morphisms, dual vector spaces, and tensor products. The following topics are discussed : - Groups and morphisms. - Quotient groups and isomorphism theorems. - Cyclic and dihedral groups. - Group actions. - Symmetric group. - Dual vector space. - Canonical form of a matrix - Tensor products and wedge products. - Tensor spaces.
Inline resources	Moodle website. Course notes, exercise sheets and projects are gradually posted during the course of the semester.
Bibliography	A. Beardon. Algebra and geometry. *Cambridge University Press, Cambridge,* 2005. S. Mac Lane and G. Birkhoff. Algebra. Third edition. *Chelsea Publishing Co., New York,* 1988. R. Godement. Cours d'algèbre. *Hermann, Paris*, 1963.
Faculty or entity in charge	MATH

Programmes containing this learning unit (UE)				
Program title	Acronym	Credits	Prerequisite	Learning outcomes
Minor in Mathematics	MINMATH	5		🔍
Additionnal module in Physics	APPHYS	5		🔍
Bachelor in Mathematics	MATH1BA	5		🔍