lmeca2660 — Numerical methods in fluid mechanics

2024

5.00 credits	30.0 h + 30.0 h	Q2

Teacher(s)	Winckelmans Grégoire ;
Language :	English > French-friendly
Place of the course	Louvain-la-Neuve
Main themes	• Reminder of the conservation equations in fluid mechanics; Reminder of the differents types of PDEs and of their classification. • Finite differences et numerical schemes for ODEs and discretized PDEs : consistency, stability, convergence, explicit and implicit schemes. • Case of 2-D and of 3-D flows, steady and unsteady. • Incompressible flows : formulation in velocity-pressure and formulation in vorticity-velocity (streamfunction) . • Compressible flows, including capture of discontinuities. • Structured grids, also with mapping from physical to computational space. Introduction to finite volumes approaches, and to unstructured grids. • Lagrangian vortex element method (VEM) eventually combined with the boundary element method (BEM)
Learning outcomes	**At the end of this learning unit, the student is able to :** 1 In consideration of the reference table AA of the program "Masters degree in Mechanical Engineering", this course contributes to the development, to the acquisition and to the evaluation of the following experiences of learning: • AA1.1, AA1.2, AA1.3 • AA2.3, AA2.4, AA2.5 • AA3.1, AA3.3 • AA5.1, AA5.2, AA5.6 • AA6.2, AA6.4 Enlarge the knowledge and skills of the students in numerical methods and initiate them to the numerical simulation in fluid mechanics (Computational Fluid Dynamics, CFD), the path followed focusing on the understanding of the physical problems and on their mathematical and numerical modelisation in an adequate formalism. Develop the aptitude of the student to realize numerical programs (codes) that "put to work" some of the numerical schemes presented in the course, in order to produce a complete numerical simulation of a physical problem.
Evaluation methods	The homeworks are essential to this course on numerical methods in fluid mechanics. They correspond to work that is mandatory and that must be performed during the quadrimester; each within a well-defined time period and with a given deadline for the report, that is graded. It is not possible to do, or even re-do, any of the homeworks outside of the time period that was defined for it within the quadrimester. The final exam is a written exam, with questions that can cover all parts of the course (lectures, exercice sessions, homeworks). The calculation of the final grade obtained by the student for the course is a weighted sum of the grade obtained for the homeworks (for 60 %) and of the grade obrained for the final written exam (for 40 %). It is however required to obtain at least 8/20 at the exam for that rule to apply. If the grade obtained at the exam is below 8/20, the final grade for the course is the grade of the exam.
Teaching methods	Lectures : there are typically 13 lectures in class, each of 2 hours. Sessions of exercices are also organised in class, each of 2 hours, to further develop concepts covered during the lectures and to do some applications. The students must perform two homeworks which require to program in C. These homeworks are mandatory and they must be done during the quadrimester, each with a start date and a deadline date for the report, which is graded. Depending on the amplitude of the work/effort expected, these homeworks are done alone or in teams of two students. The use of generative AI tools such as ChatGPT, Consensus, Perplexity,... to produce parts of results or reports is not allowed.
Content	Reminder of the different types of partial differential equations (PDEs) : hyperbolic, parabolic, elliptic. Systems of PDEs. Method of characteristics for hyperbolic cases and applications in ideal compressible flows.

	Discretisation using explicit finite differences, centered and decentered: obtention by Taylor series, truncation error and order. Definition of fundamental operators and obtention of finite difference stencils using operators. Implicit finite differences and compact schemes. Model convection equation in 1-D: discretisation of the convective term using centered finite differences, explicit and implicit, modal analysis and modified wavenumber: phase error (= numerical dispersion); decentered finite differences (upwinding) and amplitude error (= numerical diffusion). Case with convection velocity that varies with x. Model diffusion equation in 1-D: discretisation of the diffusion term using centered finite differences, explicit and implicit, modal analysis and modified wavenumber: amplitude error. Temporal integration schemes for discretized problems: numerical integration of ODE and system of ODEs; reminder of basic schemes and new schemes, stability analysis : explicit Euler, implicit Euler, Crank-Nicolson (= trapezoid rule), multi substeps schemes (Runge-Kutta), multi steps schemes (Leap Frog, Adams-Bashforth, Adams-Moulton), predictor-corrector schemes, Hyman scheme, 3BDF scheme, etc. Model convection-diffusion equation in 1-D and in 2-D : mesh Reynolds number, Fourier number and CFL (Courant-Friedrichs-Lewy) number, linear and non linear cases, integration schemes and stability, decentered finite differences for the convection (upwinding), ADI schemes for multi-dimensional problems. Numerical methods for incompressible flows : velocity-presure formulation : discretisation (staggered MAC mesh), imposition of boundary conditions, method of artificial evolution for steady flows, methods for unsteady flows, stability, Brinkman penalization method for case with an immersed bodies. Vorticity-velocity formulation: discretisation, obtention of the velocity field from the vorticity field, approximate boundary condition on the vorticity at the wall, method or artifical evolution for steady flows, method for unsteady flows. Hyperbolic systems in conservative form : model non-linear equation (Burgers), Euler equations for compressible flows and boundary conditions (based on the characteristics) ; explicit integation schemes (Lax-Wendroff, Richtmeyer, MacCormack), implicit integration schemes; numerical capture of discontinuities. Transformation of a computational structured domain (block) in a physical domain, and obtention of equations in a conservative form in the computational domain ; multi-blocks approach. Delta form of the discretized equations for multi-dimensional problems and generalized ADI schemes (Beam-Warming). Introduction to the method of finite volumes for unstructured grids: treatment of the convective and diffusive fluxes.
Inline resources	Moodle site of the course
Bibliography	• R.W. Hamming, « Numerical Methods for Scientists and Engineers », second ed., Dover, 1986. • J.H. Ferziger, « Numerical Methods for Engineering Applications », Wiley, 1981. • J. H. Ferziger and M. Peric, « Computational Methods for Fluid Dynamics », Springer, 1996. • R. Peyret and T.D. Taylor, « Computational Methods for Fluid Flow », Springer, 1986. • C.A. J. Fletcher, « Computational Techniques for Fluid Dynamics 1, Fundamental and General Techniques », second ed., Springer 1991. • C.A. J. Fletcher, « Computational Techniques for Fluid Dynamics 2, Specific Techniques for Different Flow Categories » second ed., Springer, 1991. • K. Srinivas and C.A.J Fletcher, « Computational Techniques for Fluid Dynamics, A Solutions Manual », Springer, 1991. • D.A. Anderson, J.C. Tannehill, R.H. Pletcher, « Computational Fluid Mechanics and Heat Transfer », Hemisphere Publishing, 1984. • D. Drikakis and W. Rider, « High-Resolution Methods for Incompressible and Low-Speed Flows », Springer, 2005.
Faculty or entity in charge	MECA

Programmes containing this learning unit (UE)

Program title	Acronym	Credits	Prerequisite	Learning outcomes
Master [120] in Biomedical Engineering	GBIO2M	5		
Master [120] in Mechanical Engineering	MECA2M	5		
Master [120] in Electro-mechanical Engineering	ELME2M	5		
Master [120] in Mathematical Engineering	MAP2M	5		
Master [120] in Energy Engineering	NRGY2M	5		